# Knowledge Representation for the Semantic Web

**Winter Quarter 2010**

**Slides 9 – 03/01/2010**

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# Slides are based on

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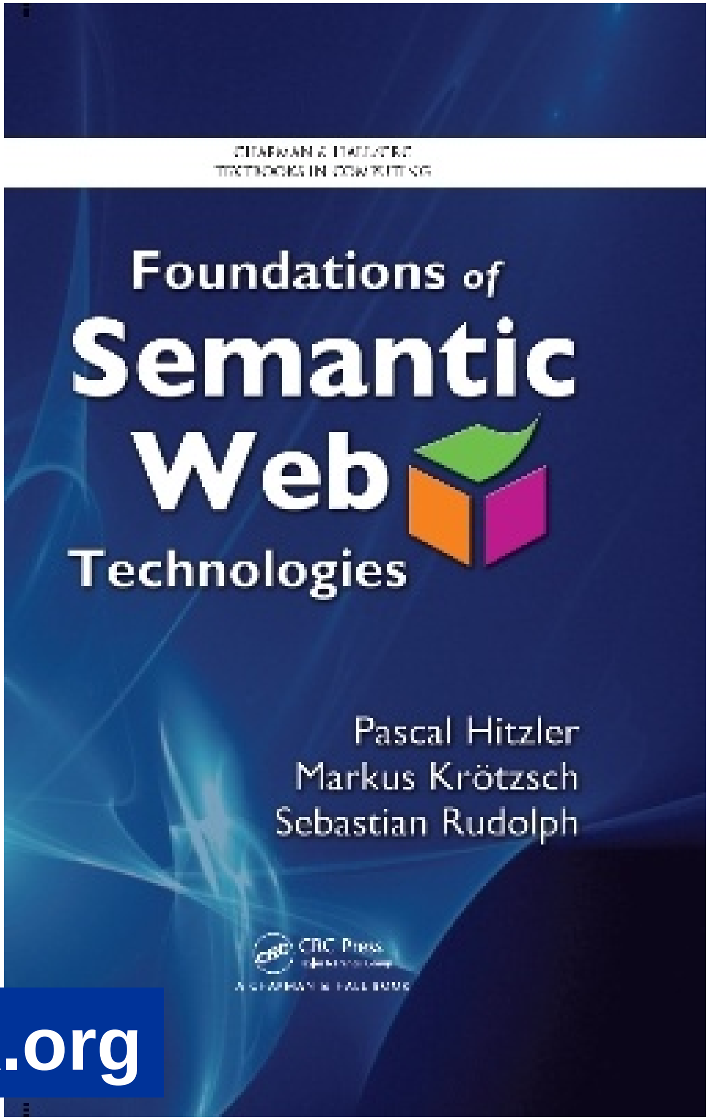
Foundations of Semantic Web Technologies

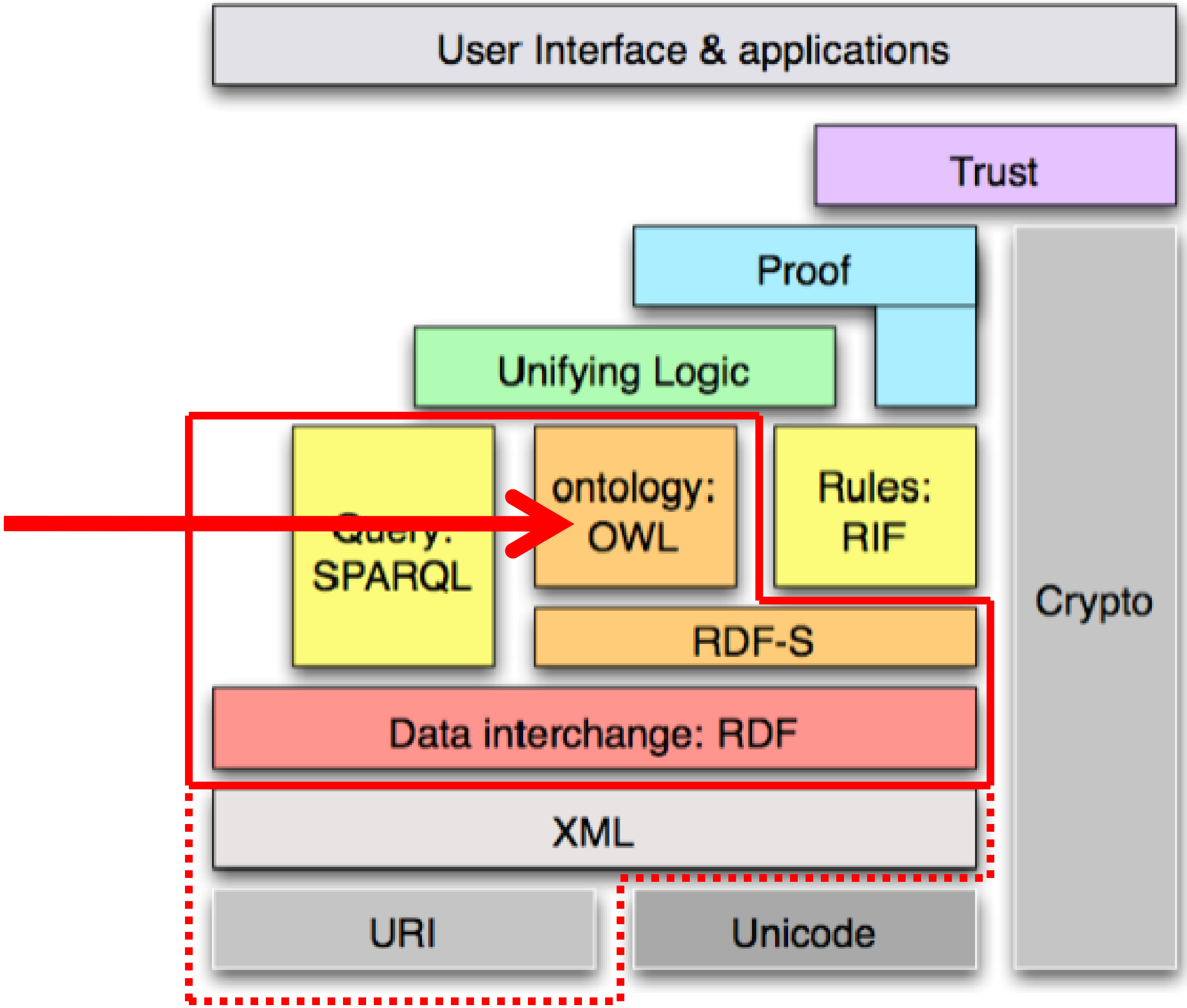
Chapman & Hall/CRC, 2010

Flyer with special offer is available.




**http://www.semantic-web-book.org**

# Today: Reasoning with OWL



# A Reasoning Problem

**A is a logical consequence of K**

**written K $\models$ A**

**if and only if**

**every model of K is a model of A.**

- **To show an entailment, we need to check all models?**
- **But that‘s infinitely many!!!**

# A Reasoning Problem

We need algorithms which do not apply the model-based definition of logical consequence in a naive manner.

These algorithms should be syntax-based.
(Computers can only do syntax manipulations.)

Luckily, such algorithms exist!

However, their correctness (soundness and completeness) needs to be proven formally.
Which is often a non-trivial problem requiring substantial mathematical build-up.

We won‘t do the proofs here.

# Contents

- Important inference problems
- Tableaux algorithm for ALC
- Tableaux algorithm for SHIQ

# Important Inference Problems

- Global consistency of a knowledge base. $KB \models \text{false}$?
  - Is the knowledge base meaningful?
- Class consistency $C \equiv \bot$?
  - Is C necessarily empty?
- Class inclusion (Subsumption) $C \sqsubseteq D$?
  - Structuring knowledge bases
- Class equivalence $C \equiv D$?
  - Are two classes in fact the same class?
- Class disjointness $C \sqcap D = \bot$?
  - Do they have common members?
- Class membership $C(a)$?
  - Is a contained in C?
- Instance Retrieval „find all x with C(x)“
  - Find all (known!) individuals belonging to a given class.

# Reduction to Unsatisfiability

- **Global consistency of a knowledge base.** — **KB unsatisfiable**
  - **Failure to find a model.**
- **Class consistency** — $C \equiv \bot$?
  - **KB $\cup$ {C(a)} unsatisfiable**
- **Class inclusion (Subsumption)** — $C \sqsubseteq D$?
  - **KB $\cup$ {$C \sqcap \neg D(a)$} unsatisfiable (a new)**
- **Class equivalence** — $C \equiv D$?
  - **$C \sqsubseteq D$ und $D \sqsubseteq C$**
- **Class disjointness** — $C \sqcap D = \bot$?
  - **KB $\cup$ {$(C \sqcap D)(a)$} unsatisfiable (a new)**
- **Class membership** — C(a)?
  - **KB $\cup$ {$\neg$C(a)} unsatisfiable**
- **Instance Retrieval** — **„find all x with C(x)“**
  - **Check class membership for all individuals.**

# Reduction to Satisfiability

- We will present so-called tableaux algorithms.

- They attempt to construct a model of the knowledge base in a „general, abstract“ manner.
  - If the construction fails, then (provably) there is no model – i.e. the knowledge base is unsatisfiable.
  - If the construction works, then it is satisfiable.

$\rightarrow$ Hence the reduction of all inference problems to the checking of unsatisfiability of the knowledge base!

# Contents

- Important inference problems
- Tableaux algorithm for ALC
- Tableaux algorithm for SHIQ

# ALC tableaux: contents

- Transformation to negation normal form
- Naive tableaux algorithm
- Tableaux algorithm with blocking

# Transform. to negation normal form

Given a knowledge base K.

- Replace $C \equiv D$ by $C \sqsubseteq D$ and $D \sqsubseteq C$.
- Replace $C \sqsubseteq D$ by $\neg C \sqcup D$.
- Apply the equations on the next slide exhaustively.

Resulting knowledge base: NNF(K)

*Negation normal form* of K.

Negation occurs only directly in front of atomic classes.

$$
\begin{aligned}
\mathrm{NNF}(C) &= C \qquad \text{if } C \text{ is a class name}\\
\mathrm{NNF}(\neg C) &= \neg C \qquad \text{if } C \text{ is a class name}\\
\mathrm{NNF}(\neg\neg C) &= \mathrm{NNF}(C)\\
\mathrm{NNF}(C \sqcup D) &= \mathrm{NNF}(C) \sqcup \mathrm{NNF}(D)\\
\mathrm{NNF}(C \sqcap D) &= \mathrm{NNF}(C) \sqcap \mathrm{NNF}(D)\\
\mathrm{NNF}(\neg(C \sqcup D)) &= \mathrm{NNF}(\neg C) \sqcap \mathrm{NNF}(\neg D)\\
\mathrm{NNF}(\neg(C \sqcap D)) &= \mathrm{NNF}(\neg C) \sqcup \mathrm{NNF}(\neg D)\\
\mathrm{NNF}(\forall R.C) &= \forall R.\mathrm{NNF}(C)\\
\mathrm{NNF}(\exists R.C) &= \exists R.\mathrm{NNF}(C)\\
\mathrm{NNF}(\neg\forall R.C) &= \exists R.\mathrm{NNF}(\neg C)\\
\mathrm{NNF}(\neg\exists R.C) &= \forall R.\mathrm{NNF}(\neg C)
\end{aligned}
$$

**K and NNF(K) have the same models (are *logically equivalent*).**

# Example

$P \sqsubseteq (E \sqcap U) \sqcup \neg(\neg E \sqcup D).$

**In negation normal form:**

$\neg P \sqcup (E \sqcap U) \sqcup (E \sqcap \neg D).$

# ALC tableaux: contents

- Transformation to negation normal form
- Naive tableaux algorithm
- Tableaux algorithm with blocking

# Naive tableaux algorithm

**Reduction to (un)satisfiability.**

**Idea:**

- **Given knowledge base K**
- **Attempt construction of a tree (called *Tableau*), which represents a model of K.
(It‘s actually rather a *Forest*.)**
- **If attempt fails, K is unsatisfiable.**

# The Tableau

- **Nodes represent elements of the domain of the model**
  **$\rightarrow$ Every node x is labeled with a set L(x) of class expressions.**
  **C $\in$ L(x) means: "x is in the extension of C"**

- **Edges stand for role relationships:**
  **$\rightarrow$ Every edge <x,y> is labeled with a set L(<x,y>) of role names.**
  **R $\in$ L(<x,y>) means: "(x,y) is in the extension of R"**

# Simple example

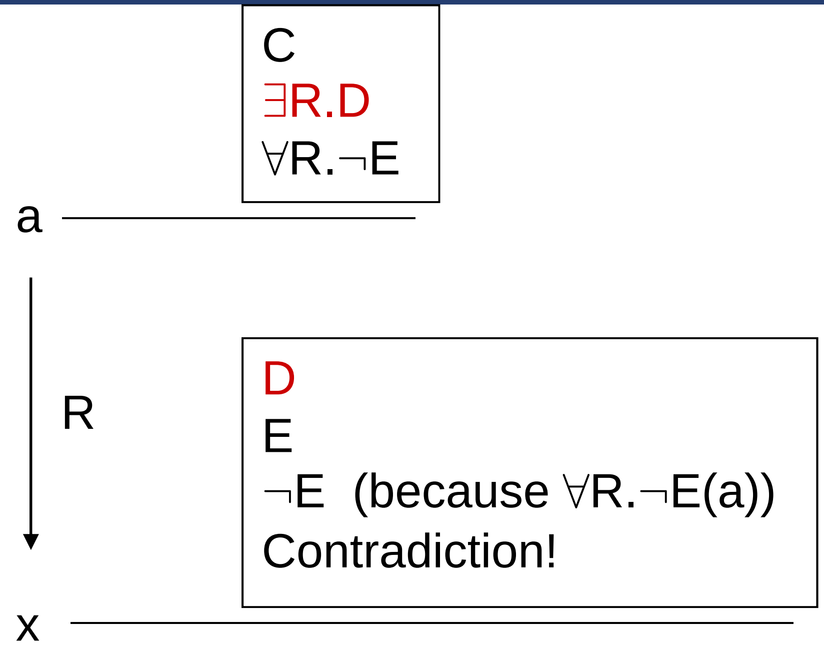
**C(a)**

**$C \sqsubseteq \exists R.D$**

**$D \sqsubseteq E$**

**Does this entail $(\exists R.E)(a)$?**

**(add $\forall R.\neg E(a)$ and show unsatisfiability)**



# Another example

**C(a)**

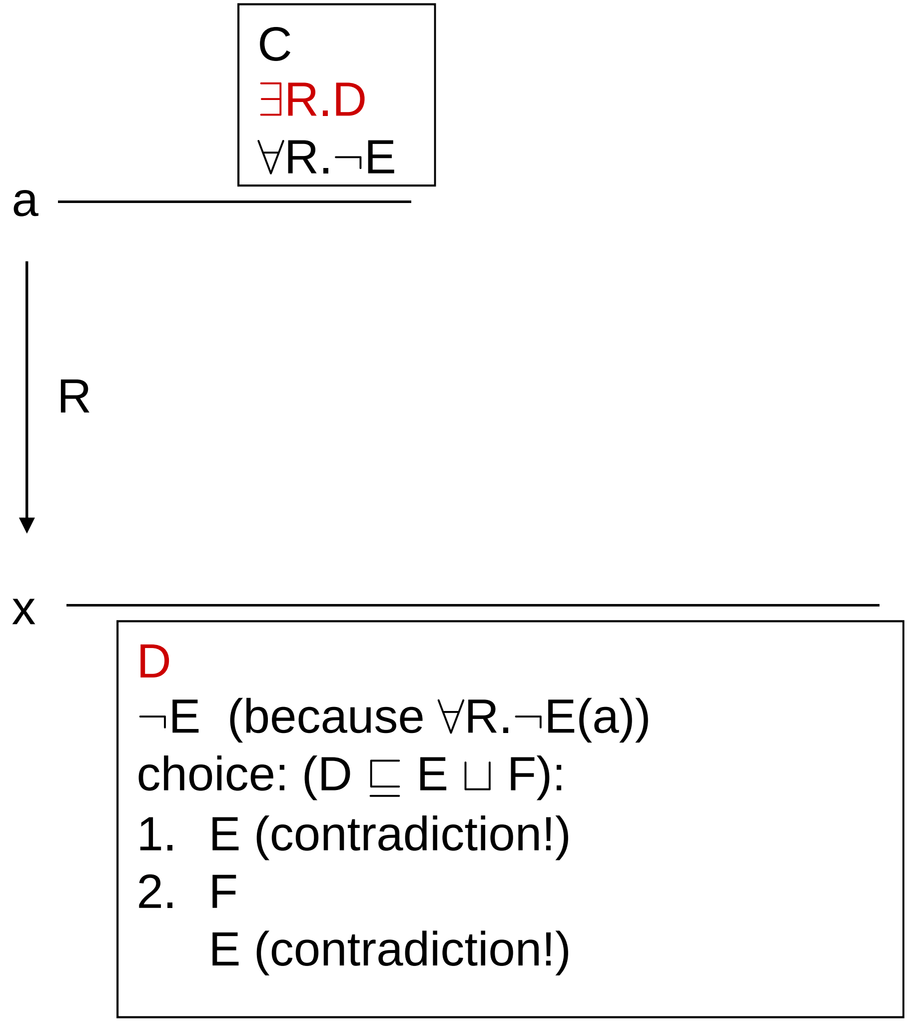
**$C \sqsubseteq \exists R.D$**

**$D \sqsubseteq E \sqcup F$**

**$F \sqsubseteq E$**

**Does this entail $(\exists R.E)(a)$?**

**(add $\forall R.\neg E(a)$ and show unsatisfiability)**



# Formal Definition

- Input: K=TBox + ABox (in NNF)
- Output: Whether or not K is satisfiable.

- A tableau is a directed labeled graph
  - nodes are individuals or (new) variable names
  - nodes x are labeled with sets L(x) of classes
  - edges <x,y> are labeled with sets L(<x,y>) of role names

# Initialisation

- **Make a node for every individual in the ABox.**
- **Every node is labeled with the corresponding class names from the ABox.**
- **There is an edge, labeled with R, between a and b, if R(a,b) is in the ABox.**

- **(If there is no ABox, the initial tableau consists of a node x with empty label.)**

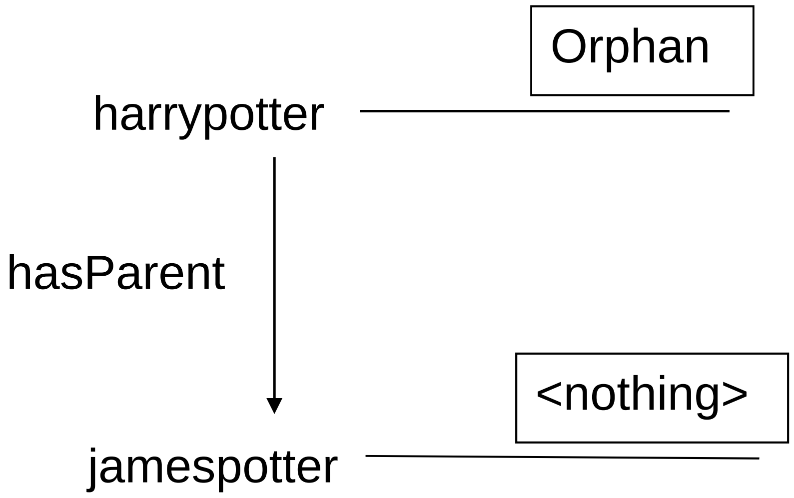
# Example initialisation

$\text{Human} \sqsubseteq \exists \text{hasParent.Human}$

$\text{Orphan} \sqsubseteq \text{Human} \sqcap \neg\exists \text{hasParent.Alive}$

**Orphan(harrypotter)**

**hasParent(harrypotter,jamespotter)**



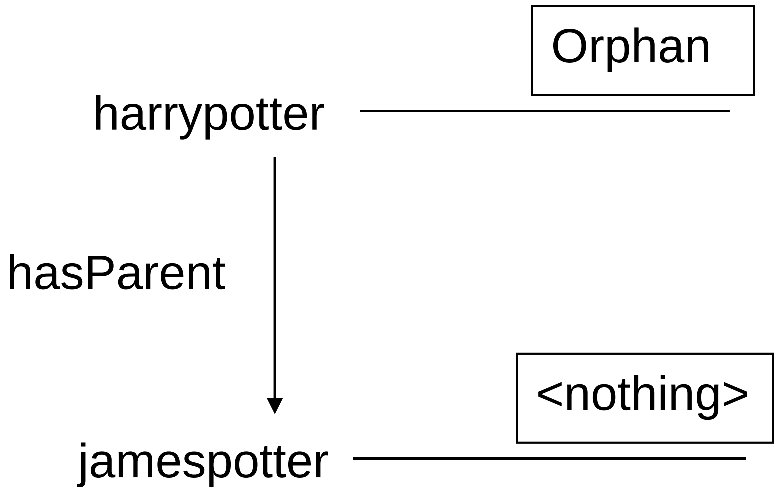
# Careful: need NNF!

$\neg\textbf{Human} \sqcup \exists\textbf{hasParent.Human}$

$\neg\textbf{Orphan} \sqcup (\textbf{Human} \sqcap \forall\textbf{hasParent.}\neg\textbf{Alive})$

**Orphan(harrypotter)**

**hasParent(harrypotter,jamespotter)**



# Constructing the tableau

- **Non-deterministically extend the tableau using the rules on the next slide.**

- **Terminate, if**
  - **there is a contradiction in a node label (i.e., it contains classes C and ¬C, or it contains ⊥), or**
  - **none of the rules is applicable.**

- **If the tableau does not contain a contradiction, then the knowledge base is satisfiable.**
  **Or more precisely: If you can make a choice of rule applications such that no contradiction occurs and the process terminates, then the knowledge base is satisfiable.**

# Naive ALC tableaux rules

**$\sqcap$-rule:** If $C \sqcap D \in \mathcal{L}(x)$ and $\{C, D\} \not\subseteq \mathcal{L}(x)$, then set $\mathcal{L}(x) \leftarrow \{C, D\}$.

**$\sqcup$-rule:** If $C \sqcup D \in \mathcal{L}(x)$ and $\{C, D\} \cap \mathcal{L}(x) = \emptyset$, then set $\mathcal{L}(x) \leftarrow C$ or $\mathcal{L}(x) \leftarrow D$.

**$\exists$-rule:** If $\exists R.C \in \mathcal{L}(x)$ and there is no $y$ with $R \in L(x, y)$ and $C \in \mathcal{L}(y)$, then

1. add a new node with label $y$ (where $y$ is a new node label),
2. set $\mathcal{L}(x, y) = \{R\}$, and
3. set $\mathcal{L}(y) = \{C\}$.

**$\forall$-rule:** If $\forall R.C \in \mathcal{L}(x)$ and there is a node $y$ with $R \in \mathcal{L}(x, y)$ and $C \notin \mathcal{L}(y)$, then set $\mathcal{L}(y) \leftarrow C$.

**TBox-rule:** If $C$ is a TBox statement and $C \notin \mathcal{L}(x)$, then set $\mathcal{L}(x) \leftarrow C$.

# Example

$\neg$Alive(jamespotter)
i.e. add: Alive(jamespotter)
and search for contradiction

**$\neg$Human $\sqcup$ $\exists$hasParent.Human**

**$\neg$Orphan $\sqcup$ (Human $\sqcap$ $\forall$hasParent.$\neg$Alive)**

**Orphan(harrypotter)**

**hasParent(harrypotter,jamespotter)**

harrypotter

Orphan
$\neg$Orphan $\sqcup$ (Human $\sqcap$ $\forall$hasParent.$\neg$Alive)
1. $\neg$Orphan (contradiction)
2. Human $\sqcap$ $\forall$hasParent.$\neg$Alive
   Human
   $\forall$hasParent.$\neg$Alive

hasParent

Alive
2. $\neg$Alive (contradiction)

jamespotter

# ALC tableaux: contents

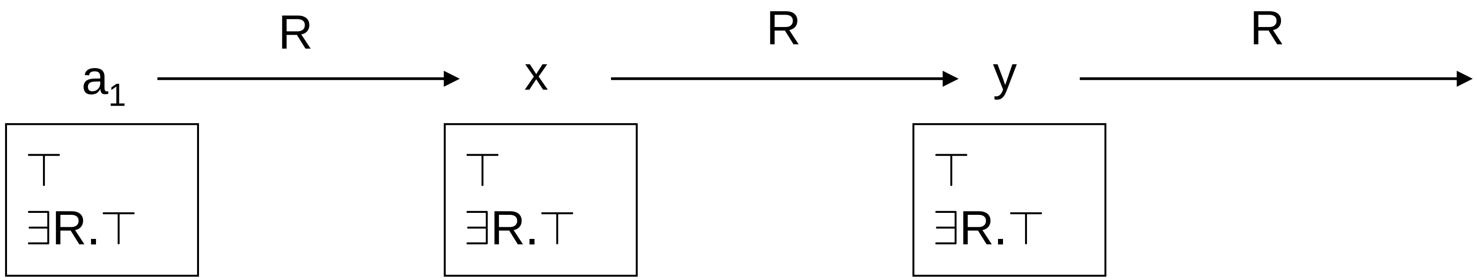
- **Transformation to negation normal form**
- **Naive tableaux algorithm**
- **Tableaux algorithm with blocking**

# There‘s a termination problem

**TBox: $\exists R.\top$**

**ABox: $\top(a_1)$**

- **Obviously satisfiable:**
  **Model M with domain elements $a_1^M, a_2^M, \ldots$**
  **and $R^M(a_i^M, a_{i+1}^M)$ for all $i \geq 1$**
- **but tableaux algorithm does not terminate!**

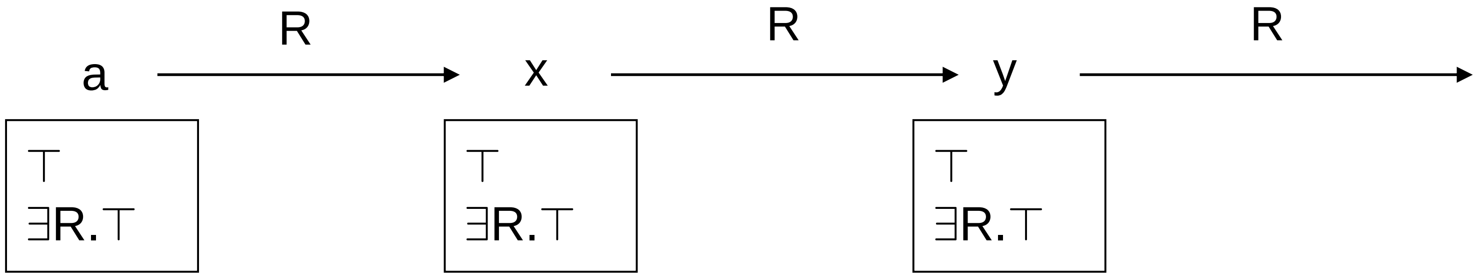


# Solution?

**Actually, things repeat!**

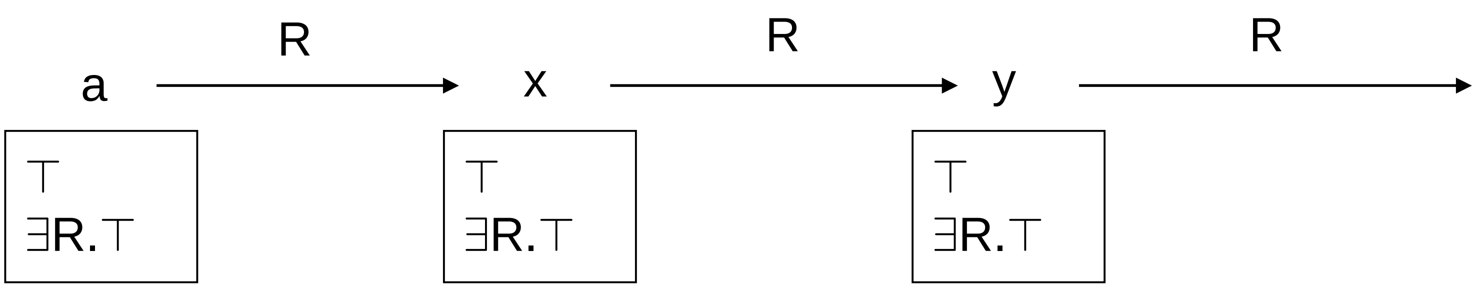
**Idea: it is not necessary to expand x, since it‘s simply a copy of a.**

$\Rightarrow$ **Blocking**



# Blocking

- x is *blocked* (by y) if
  - x is not an individual (but a variable)
  - y is a predecessor of x and $L(x) \subseteq L(y)$
  - or a predecessor of x is blocked




**Here, x is blocked by a.**

# Constructing the tableau

- **Non-deterministically extend the tableau using the rules on the next slide, but only apply a rule if x is not blocked!**

- **Terminate, if**
  - **there is a contradiction in a node label (i.e., it contains classes C and $\neg$C), or**
  - **none of the rules is applicable.**

- **If the tableau does not contain a contradiction, then the knowledge base is satisfiable.**
  **Or more precisely: If you can make a choice of rule applications such that no contradiction occurs and the process terminates, then the knowledge base is satisfiable.**

# Naive ALC tableaux rules

**$\sqcap$-rule:** If $C \sqcap D \in \mathcal{L}(x)$ and $\{C, D\} \not\subseteq \mathcal{L}(x)$, then set $\mathcal{L}(x) \leftarrow \{C, D\}$.

**$\sqcup$-rule:** If $C \sqcup D \in \mathcal{L}(x)$ and $\{C, D\} \cap \mathcal{L}(x) = \emptyset$, then set $\mathcal{L}(x) \leftarrow C$ or $\mathcal{L}(x) \leftarrow D$.

**$\exists$-rule:** If $\exists R.C \in \mathcal{L}(x)$ and there is no $y$ with $R \in L(x, y)$ and $C \in \mathcal{L}(y)$, then

1. add a new node with label $y$ (where $y$ is a new node label),
2. set $\mathcal{L}(x, y) = \{R\}$, and
3. set $\mathcal{L}(y) = \{C\}$.

**$\forall$-rule:** If $\forall R.C \in \mathcal{L}(x)$ and there is a node $y$ with $R \in \mathcal{L}(x, y)$ and $C \notin \mathcal{L}(y)$, then set $\mathcal{L}(y) \leftarrow C$.

**TBox-rule:** If $C$ is a TBox statement and $C \notin \mathcal{L}(x)$, then set $\mathcal{L}(x) \leftarrow C$.

**Apply only if x is not blocked!**

# Example (0)

- **Knowledge base {Human ⊑ ∃hasParent.Human, Bird(tweety)}**
- **We want to show that Human(tweety) does *not* hold,
  i.e. that ¬Human(tweety) is entailed.**
- **We will not be able to show this.
  I.e. Human(tweety) is *possible*.**

- **Shorter notation:**
  **H ⊑ ∃p.H**
  **B(t)**

  **¬H(t) entailed?**

# Example (0)

**Knowledge base {¬H ⊔ ∃p.H, B(t), H(t)}**

t —p→ x

H
B
¬H ⊔ ∃p.H
1. ¬H (contradiction)
2. ∃p.H

2.:
H
blocked by t!

**expansion stops. Cannot find contradiction!**

# Example (0) the other case

**Knowledge base {¬H ⊔ ∃p.H, B(t), ¬H(t)}**

t —p→ x —p→ y

**t:**

¬H
B
¬H ⊔ ∃p.H
1. ¬H cannot be added. no expansion in this part
2. ∃p.H

**x:**

2.:
H
¬H ⊔ ∃p.H
2.1: ¬H (contradiction)
2.2: ∃p.H

**y:**

2.2:
H
blocked by x

**no further expansion possible – knowledge base is satisfiable!**

# Example(1)

**Show, that**

**Professor $\sqsubseteq$ (Person $\sqcap$ Unversitymember)**
**$\sqcup$ (Person $\sqcap$ $\neg$PhDstudent)**

**entails that every Professor is a Person.**

Find contradiction in:
$\neg P \sqcup (E \sqcap U) \sqcup (E \sqcap \neg S)$
$P \sqcap \neg E(x)$

x

$P \sqcap \neg E$
$P$
$\neg E$
$\neg P \sqcup (E \sqcap U) \sqcup (E \sqcap \neg S)$
1. $\neg P$ (contradiction)
2. $(E \sqcap U) \sqcup (E \sqcap \neg S)$
    1. $E \sqcap U$
       $E$ (contradiction)
    2. $E \sqcap \neg S$
       $E$ (contradiction)

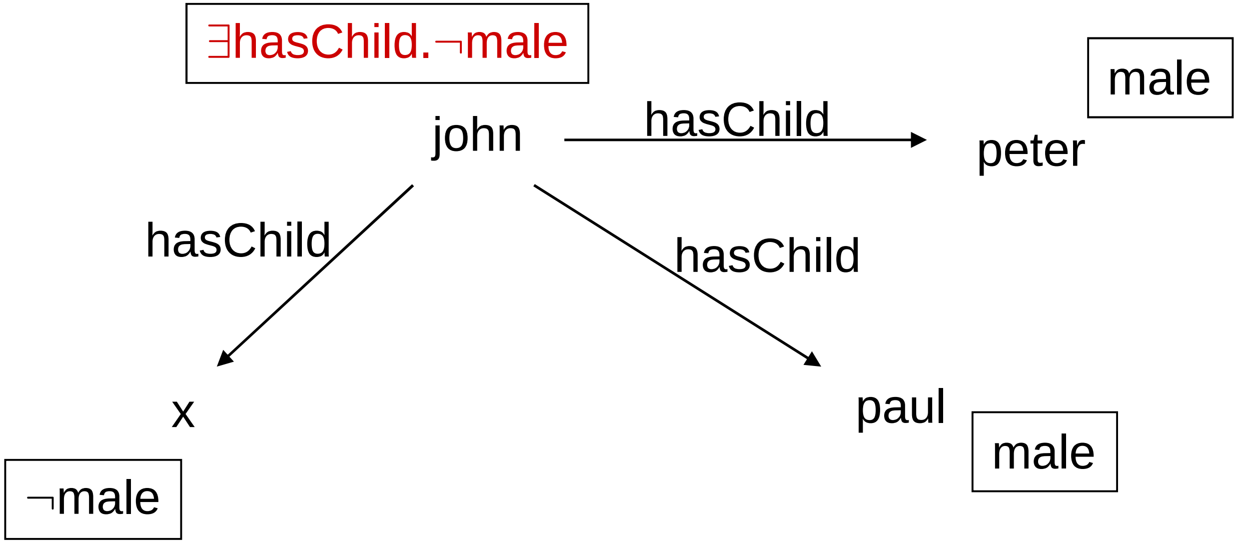
# Example (2)

**Show that**

**hasChild(john, peter)**
**hasChild(john, paul)**
**male(peter)**
**male(paul)**

**does *not* entail ∀hasChild.male(john).**

¬∀hasChild.male ≡ ∃hasChild.¬male



# Example (3)

**Show that the knowledge base**

**Bird ⊑ Flies**

**Penguin ⊑ Bird**

**Penguin ⊓ Flies ⊑ ⊥**

**Penguin(tweety)**

**is unsatisfiable.**

TBox:
¬B ⊔ F
¬P ⊔ B
¬P ⊔ ¬F ⊔ ⊥

tweety

P
¬P ⊔ B
¬B ⊔ F
¬P ⊔ ¬F

1. ¬P (contradiction)
2. B
    1. ¬B (contradiction)
    2. F
        1. ¬P (contradiction)
        2. ¬F (contradiction)

# Example (4)

Show that the knowledge base

C(a) C(c)

R(a,b) R(a,c)

S(a,a) S(c,b)

$C \sqsubseteq \forall S.A$

$A \sqsubseteq \exists R.\exists S.A$

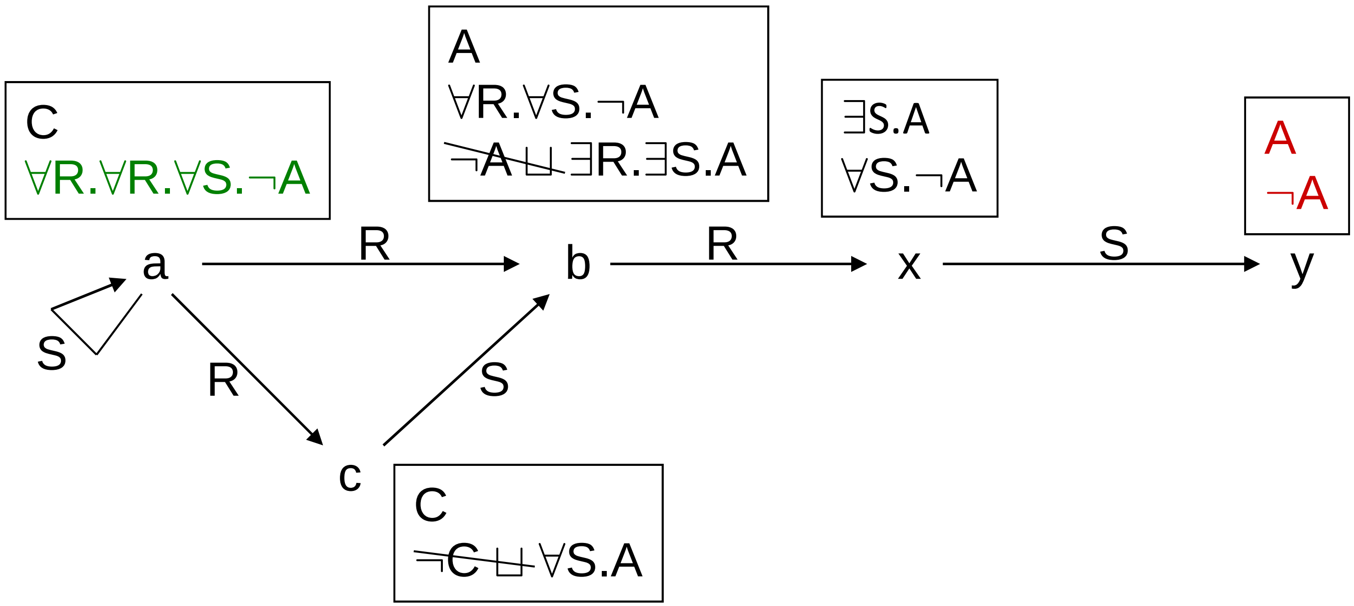
$A \sqsubseteq \exists R.C$

entails $\exists R.\exists R.\exists S.A(a)$.

# Example (4)

$\neg\exists R.\exists R.\exists S.A \equiv \forall R.\forall R.\forall S.\neg A$

TBox:
$\neg C \sqcup \forall S.A$
$\neg A \sqcup \exists R.\exists S.A$
$\neg A \sqcup \exists R.C$



# Contents

- Important inference problems
- Tableaux algorithm for ALC
- Tableaux algorithm for SHIQ

# Tableaux Algorithm for SHIQ

- **Basic idea is the same.**
- **Blocking rule is more complicated**
- **Other modifictions are also needed.**

# Transform. to negation normal form

Given a knowledge base K.

- Replace $C \equiv D$ by $C \sqsubseteq D$ and $D \sqsubseteq C$.
- Replace $C \sqsubseteq D$ by $\neg C \sqcup D$.
- Apply the equations on the next slide exhaustively.

Resulting knowledge base: NNF(K)

*Negation normal form* of K.

Negation occurs only directly in front of atomic classes.

$$\mathrm{NNF}(C) = C \qquad \text{if } C \text{ is a class name}$$
$$\mathrm{NNF}(\neg C) = \neg C \qquad \text{if } C \text{ is a class name}$$
$$\mathrm{NNF}(\neg\neg C) = \mathrm{NNF}(C)$$
$$\mathrm{NNF}(C \sqcup D) = \mathrm{NNF}(C) \sqcup \mathrm{NNF}(D)$$
$$\mathrm{NNF}(C \sqcap D) = \mathrm{NNF}(C) \sqcap \mathrm{NNF}(D)$$
$$\mathrm{NNF}(\neg(C \sqcup D)) = \mathrm{NNF}(\neg C) \sqcap \mathrm{NNF}(\neg D)$$
$$\mathrm{NNF}(\neg(C \sqcap D)) = \mathrm{NNF}(\neg C) \sqcup \mathrm{NNF}(\neg D)$$
$$\mathrm{NNF}(\forall R.C) = \forall R.\mathrm{NNF}(C)$$
$$\mathrm{NNF}(\exists R.C) = \exists R.\mathrm{NNF}(C)$$
$$\mathrm{NNF}(\neg\forall R.C) = \exists R.\mathrm{NNF}(\neg C)$$
$$\mathrm{NNF}(\neg\exists R.C) = \forall R.\mathrm{NNF}(\neg C)$$

**NNF($\leq$n R.C) = $\leq$n R.NNF(C)**
**NNF($\geq$n R.C) = $\geq$n R.NNF(C)**
**NNF($\neg$ $\leq$n R.C) = $\geq$(n+1)R.NNF(C)**
**NNF($\neg$ $\geq$n R.C) = $\leq$(n-1)R.NNF(C), where $\leq$(-1)R.C = $\bot$**

**K and NNF(K) have the same models (are *logically equivalent*).**

# Formal Definition

- A tableau is a directed labeled graph
  - nodes are individuals or (new) variable names
  - nodes x are labeled with sets L(x) of classes
  - edges <x,y> are labeled
    - either with sets L(<x,y>) of role names or inverse role names
    - or with the symbol = (for equality)
    - or with the symbol ≠ (for inequality)

# Initialisation

- **Make a node for every individual in the ABox. These nodes are called *root nodes*.**
- **Every node is labeled with the corresponding class names from the ABox.**
- **There is an edge, labeled with R, between a and b, if R(a,b) is in the ABox.**
- **There is an edge, labeled ≠, between a and b if a ≠ b is in the ABox.**
- **There are no = relations (yet).**

# Notions

- We write $S^{--}$ as S.
- If R $\in$ L(<x,y>) and R $\sqsubseteq$ S (where R,S can be inverse roles), then
  - y is an S-successor of x and
  - x is an S-predecessor of y.
- If y is an S-successor or an $S^{-}$-predecessor of x, then y is an *neighbor* of x.
- *Ancestor* is the transitive closure of *Predecessor.*

# Blocking for SHIQ

- x is *blocked* by y if x,y are not root nodes and
  - the following hold: ["x is directly blocked"]
    - no ancestor of x is blocked
    - there are predecessors y', x' of x
    - y is a successor of y' and x is a successor of x'
    - L(x) = L(y) and L(x') = L(y')
    - L(<x',x>) = L(<y',y>)
  - or the following holds: ["x is indirectly blocked"]
    - an ancestor of x is blocked or
    - x is successor of some y with L(<y,x>) = ∅

# Constructing the tableau

- **Non-deterministically extend the tableau using the rules on the next slide.**
- **Terminate, if**
  - **there is a contradiction in a node label, i.e.,**
    - **it contains $\perp$ or classes C and $\neg$C or**
    - **it contains a class $\leq$ nR.C and**
      **x also has (n+1) R-successors $y_i$ and $y_i \neq y_j$ (for all $i \neq j$)**
  - **or none of the rules is applicable.**
- **If the tableau does not contain a contradiction, then the knowledge base is satisfiable.**
  **Or more precisely: If you can make a choice of rule applications such that no contradiction occurs and the process terminates, then the knowledge base is satisfiable.**

# SHIQ Tableaux Rules

**$\sqcap$-rule:** If $x$ is not indirectly blocked, $C \sqcap D \in \mathcal{L}(x)$, and $\{C, D\} \not\subseteq \mathcal{L}(x)$, then set $\mathcal{L}(x) \leftarrow \{C, D\}$.

**$\sqcup$-rule:** If $x$ is not indirectly blocked, $C \sqcup D \in \mathcal{L}(x)$ and $\{C, D\} \sqcap \mathcal{L}(x) = \emptyset$, then set $\mathcal{L}(x) \leftarrow C$ or $\mathcal{L}(x) \leftarrow D$.

$\exists$-rule: If $x$ is not blocked, $\exists R.C \in \mathcal{L}(x)$, and there is no $y$ with $R \in \mathcal{L}(x, y)$ and $C \in \mathcal{L}(y)$, then

1. add a new node with label $y$ (where $y$ is a new node label),
2. set $\mathcal{L}(x, y) = \{R\}$ and $\mathcal{L}(y) = \{C\}$.

**$\forall$-rule:** If $x$ is not indirectly blocked, $\forall R.C \in \mathcal{L}(x)$, and there is a node $y$ with $R \in \mathcal{L}(x, y)$ and $C \notin \mathcal{L}(y)$, then set $\mathcal{L}(y) \leftarrow C$.

**TBox-rule:** If $x$ is not indirectly blocked, $C$ is a TBox statement, and $C \notin \mathcal{L}(x)$, then set $\mathcal{L}(x) \leftarrow C$.

# SHIQ Tableaux Rules

**trans-rule:** If $x$ is not indirectly blocked, $\forall S.C \in \mathcal{L}(x)$, $S$ has a transitive subrole $R$, and $x$ has an $R$-neighbor $y$ with $\forall R.C \notin \mathcal{L}(y)$, then set $\mathcal{L}(y) \leftarrow \forall R.C$.

**choose-rule:** If $x$ is not indirectly blocked, $\leq nS.C \in \mathcal{L}(x)$ or $\geq nS.C \in \mathcal{L}(x)$, and there is an $S$-neighbor $y$ of $x$ with $\{C, \mathrm{NNF}(\neg C)\} \cap \mathcal{L}(y) = \emptyset$, then set $\mathcal{L}(y) \leftarrow C$ or $\mathcal{L}(y) \leftarrow \mathrm{NNF}(\neg C)$.

**$\geq$-rule:** If $x$ is not blocked, $\geq nS.C \in \mathcal{L}(x)$, and there are no $n$ $S$-neighbors $y_1, \ldots, y_n$ of $x$ with $C \in \mathcal{L}(y_i)$ and $y_i \not\approx y_j$ for $i, j \in \{1, \ldots, n\}$ and $i \neq j$, then

1. create $n$ new nodes with labels $y_1, \ldots, y_n$ (where the labels are new),
2. set $\mathcal{L}(x, y_i) = \{S\}$, $\mathcal{L}(y_i) = \{C\}$, and $y_i \not\approx y_j$ for all $i, j \in \{1, \ldots, n\}$ with $i \neq j$.

**$\leq$-rule:** If $x$ is not indirectly blocked, $\leq nS.C \in \mathcal{L}(x)$, there are more than $n$ $S$-neighbors $y_i$ of $x$ with $C \in \mathcal{L}(y_i)$, and $x$ has two $S$-neighbors $y, z$ such that $y$ is neither a root node nor an ancestor of $z$, $y \not\approx z$ does not hold, and $C \in \mathcal{L}(y) \cap \mathcal{L}(z)$, then

1. set $\mathcal{L}(z) \leftarrow \mathcal{L}(y)$,
2. if $z$ is an ancestor of $x$, then $\mathcal{L}(z, x) \leftarrow \{\text{Inv}(R) \mid R \in \mathcal{L}(x, y)\}$,
3. if $z$ is not an ancestor of $x$, then $\mathcal{L}(x, z) \leftarrow \mathcal{L}(x, y)$,
4. set $\mathcal{L}(x, y) = \emptyset$, and
5. set $u \not\approx z$ for all $u$ with $u \not\approx y$.

**$\leq$-root-rule:** If $\leq nS.C \in \mathcal{L}(x)$, there are more than $n$ $S$-neighbors $y_i$ of $x$ with $C \in \mathcal{L}(y_i)$, and $x$ has two $S$-neighbors $y, z$ which are both root nodes, $y \not\approx z$ does not hold, and $C \in \mathcal{L}(y) \cap \mathcal{L}(z)$, then

1. set $\mathcal{L}(z) \leftarrow \mathcal{L}(y)$,
2. for all directed edges from $y$ to some $w$, set $\mathcal{L}(z, w) \leftarrow \mathcal{L}(y, w)$,
3. for all directed edges from some $w$ to $y$, set $\mathcal{L}(w, z) \leftarrow \mathcal{L}(w, y)$,
4. set $\mathcal{L}(y) = \mathcal{L}(w, y) = \mathcal{L}(y, w) = \emptyset$ for all $w$,
5. set $u \not\approx z$ for all $u$ with $u \not\approx y$, and
6. set $y \approx z$.

# Example (1): cardinalities

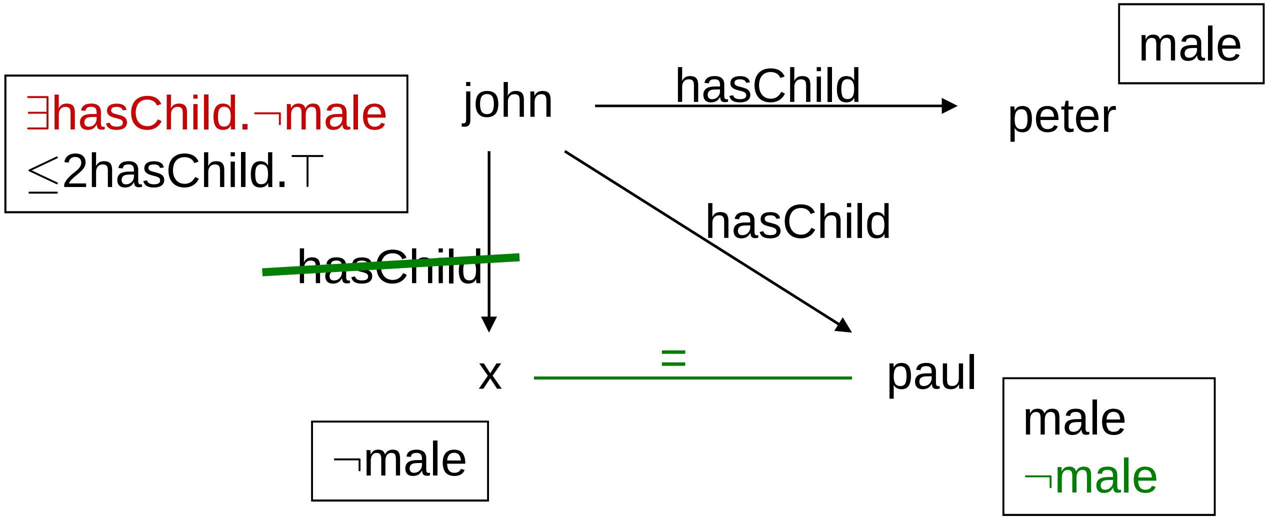
**Show, that**

**hasChild(john, peter)**
**hasChild(john, paul)**
**male(peter)**
**male(paul)**
**$\leq$2hasChild.$\top$(john)**

**does *not* entail ∀hasChild.male(john).**

¬∀hasChild.male ≡ ∃hasChild.¬male

now apply $\leq$



# Example (1): cardinalities

**Show, that**

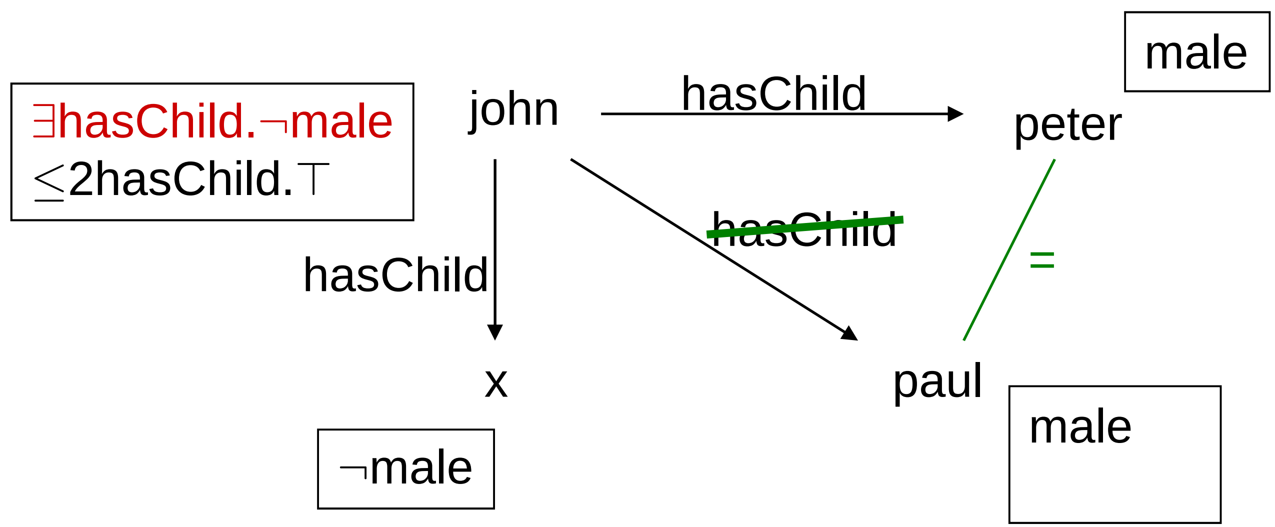
**hasChild(john, peter)**
**hasChild(john, paul)**
**male(peter)**
**male(paul)**
**$\leq$2hasChild.$\top$(john)**

**does *not* entail $\forall$hasChild.male(john).**

$\neg\forall$hasChild.male $\equiv$ $\exists$hasChild.$\neg$male

backtracking!

now apply $\leq$



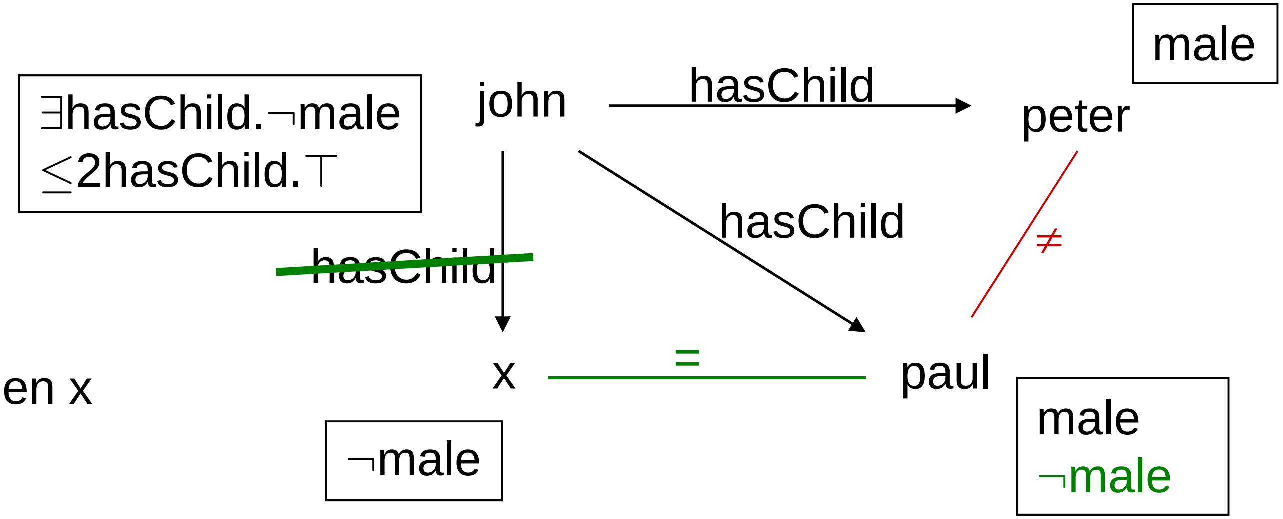
# Example (1): cardinalities – again

**Show, that**

**hasChild(john, peter)**
**hasChild(john, paul)**
**male(peter)**
**male(paul)**
**$\leq$2hasChild.$\top$(john) and peter $\neq$ paul**

**does *not* entail $\forall$hasChild.male(john).**

$\neg\forall$hasChild.male $\equiv$ $\exists$hasChild.$\neg$male




now apply $\leq$

can backtrack only between x and peter – also leads to contradiction

# Example (2): cardinalities

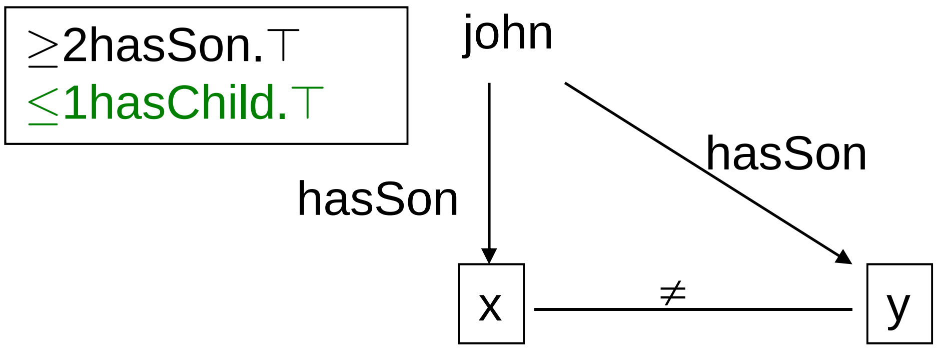
**Show, that**

**$\geq$2hasSon.$\top$(john)**

**entails $\geq$2hasChild.$\top$(john).**

$\neg\geq$2hasSon.$\top$ $\equiv$ $\leq$1hasChild.$\top$

**hasSon $\sqsubseteq$ hasChild**




hasSon-neighbors are also hasChild-neighbors, tableau terminates with contradiction

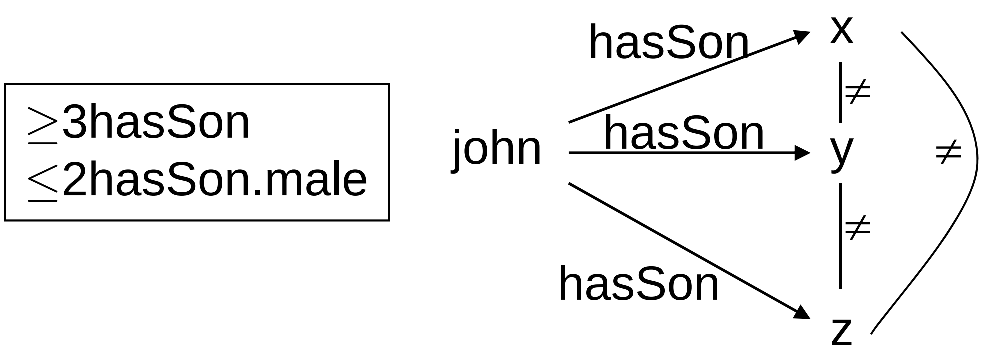
# Example (3): choose

$\geq$**3hasSon(john)**

$\leq$**2hasSon.male(john)**

**Is this contradictory?**

**No, because the following tableau is complete.**



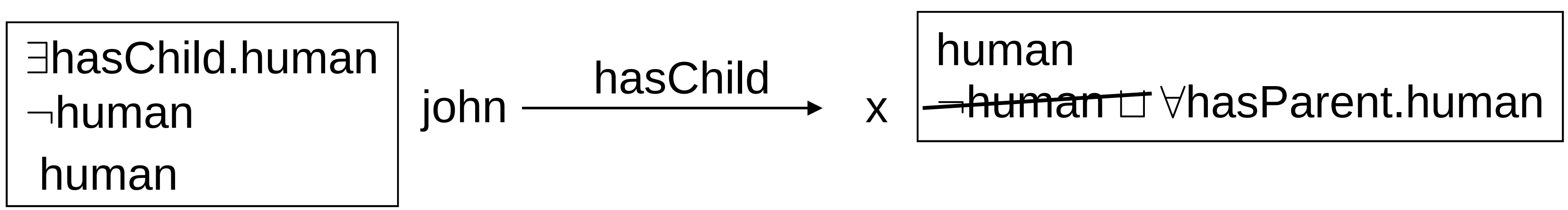
# Example (4): inverse roles

$\exists hasChild.human(john)$

$human \sqsubseteq \forall hasParent.human$

$hasChild \sqsubseteq hasParent^{-}$

**zu zeigen: human(john)**




john is $hP^{-}$-predecessor of x, hence hP-neighbor of x

# Example (5): Transitivity and Blocking

**human $\sqsubseteq \exists$hasFather.$\top$**

**human $\sqsubseteq \forall$hasAncestor.human**

**hasFather $\sqsubseteq$ hasAncestor        Trans(hasAncestor)**

**human(john)**

**Does this entail $\leq$1hasFather.$\top$(john)?**

**Negation: $\geq$2hasFather.$\top$(john)**

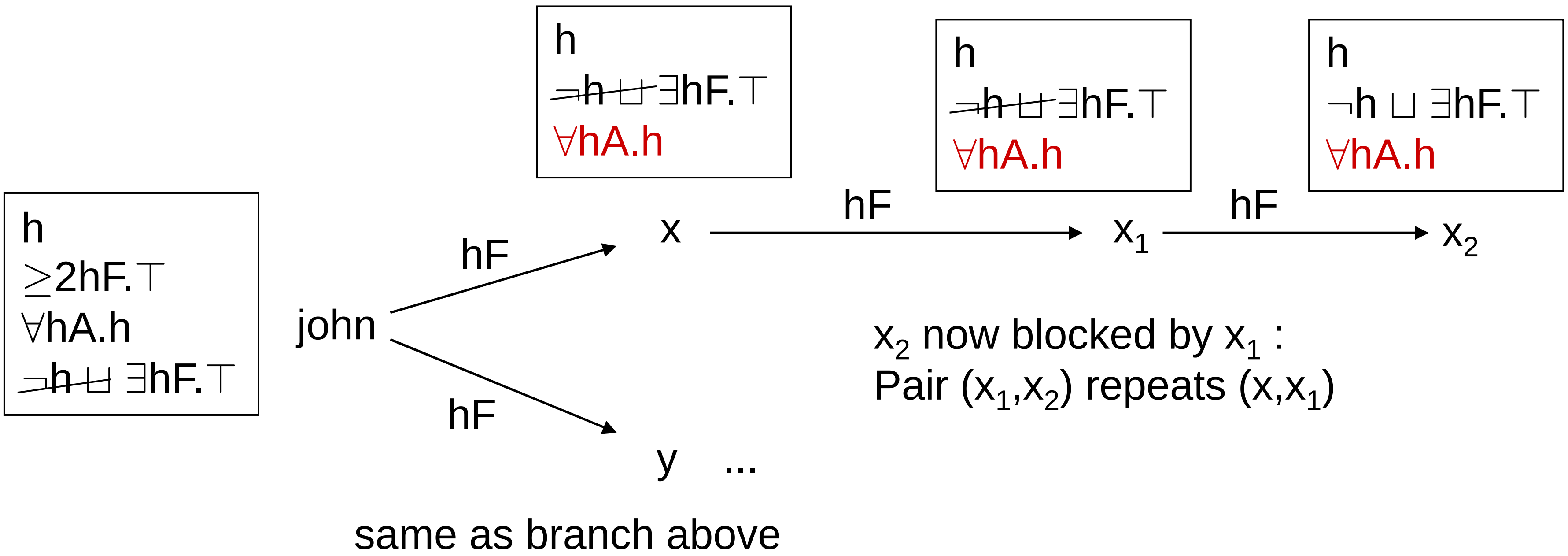
# Example (5): Transitivity and Blocking

**human ⊑ ∃hasFather.⊤**

**hasFather ⊑ hasAncestor** **Trans(hasAncestor)**

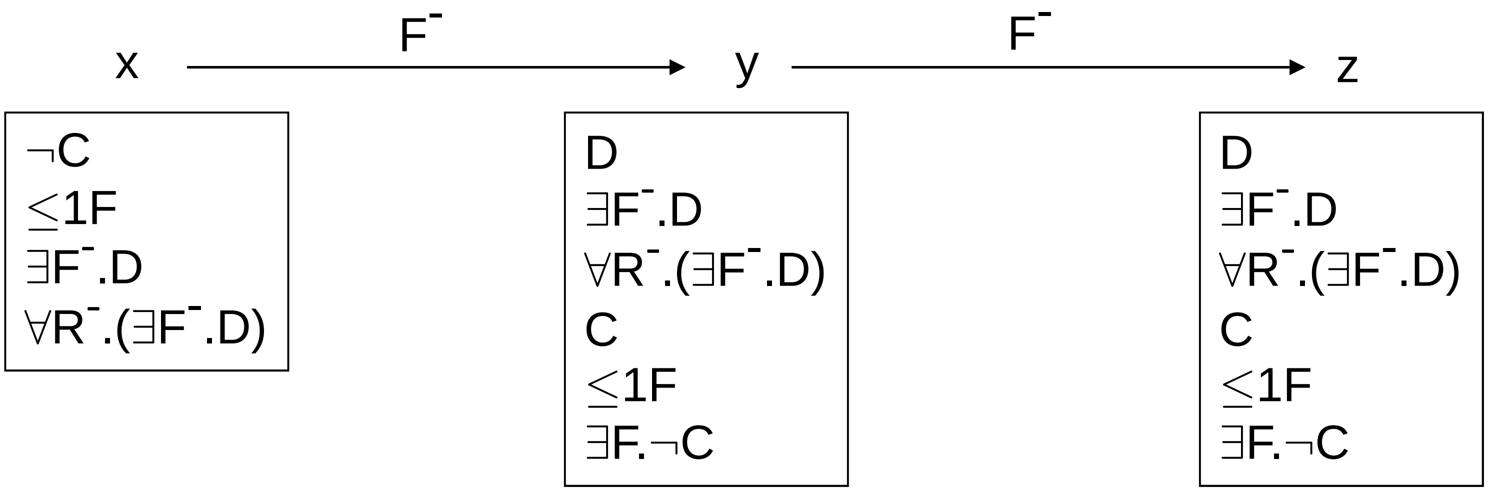
**∀hasAncestor.human(john)**

**human(john)** **≥2hasFather.⊤(john)**



# Example (6): Pairwise Blocking

**$\neg C \sqcap (\leq 1F) \sqcap \exists F^{-}.D \sqcap \forall R^{-}.(\exists F^{-}.D)$, where**
**$D = C \sqcap (\leq 1F) \sqcap \exists F.\neg C$, Trans(R), and $F \sqsubseteq R$,**
**is not satisfiable.**



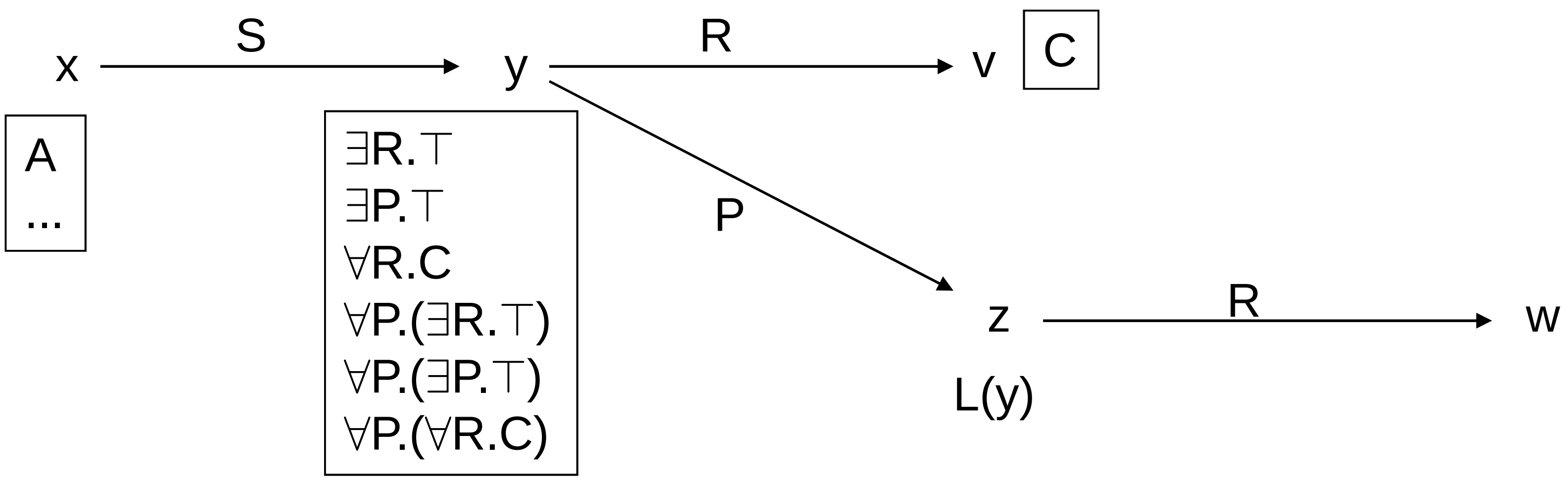

Without pairwise blocking, z would be blocked, which shouldn‘t happen:
Expansion of $\exists F.\neg C$ yields $\neg C$ for node y as required.

# Example (7): Dynamic Blocking

**$A \sqcap \exists S.(\exists R.\top \sqcap \exists P.\top \sqcap \forall R.C \sqcap \forall P.(\exists R.\top) \sqcap \forall P.(\forall R.C) \sqcap \forall P.(\exists P.\top))$**

**with $C = \forall R^-.(\forall P^-.(\forall S^-.\neg A))$ and Trans(P), is not satisfiable.**

**Part of the tableau:**




At this stage, z would be blocked by y (assuming the presence of another pair). However, when C from v is expanded, z becomes unblocked, which is necessary in order to label w with C which in turn labels x with $\neg$A, yielding the required contradiction.

# Tableaux Reasoners

- **Fact++**
  - **http://owl.man.ac.uk/factplusplus/**
- **Pellet**
  - **http://www.mindswap.org/2003/pellet/index.shtml**
- **RacerPro**
  - **http://www.sts.tu-harburg.de/~r.f.moeller/racer/**

# Tomorrow

**Please don’t forget the preparations**

**for the interactive class project session.**