Knowledge Representation for the Semantic Web

Winter Quarter 2010

Slides 7 – 02/11/2010

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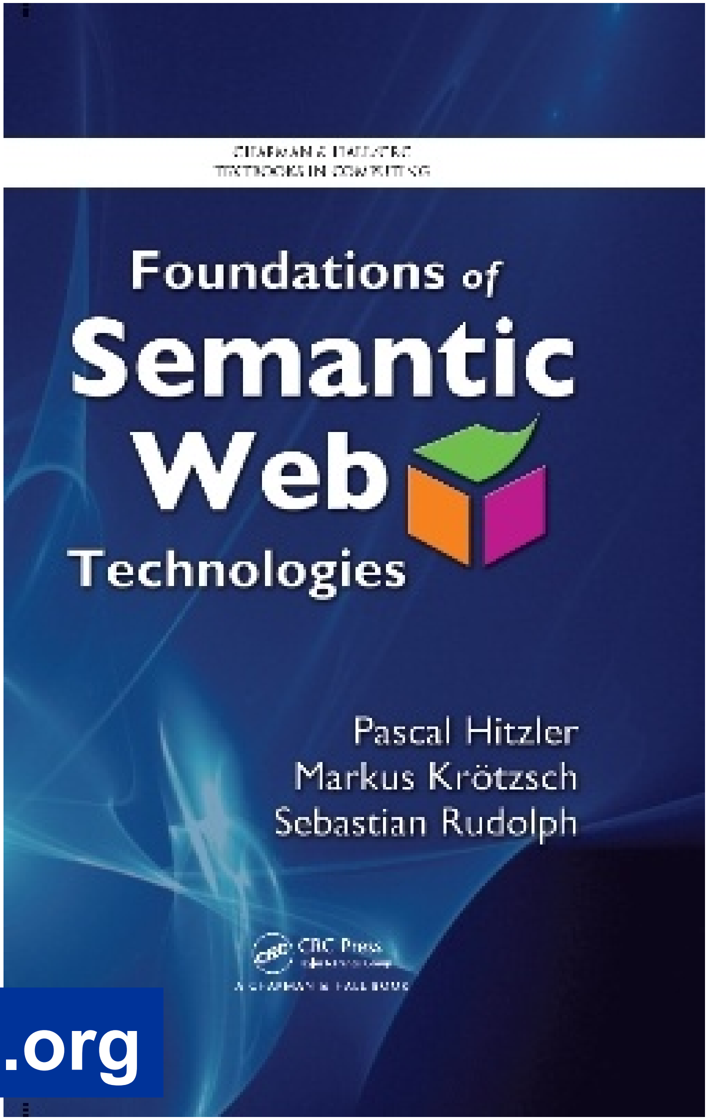
Slides are based on

Pascal Hitzler, Markus Krötzsch, Sebastian Rudolph

Foundations of Semantic Web Technologies

Chapman & Hall/CRC, 2010

Flyer with special offer is available.



http://www.semantic-web-book.org

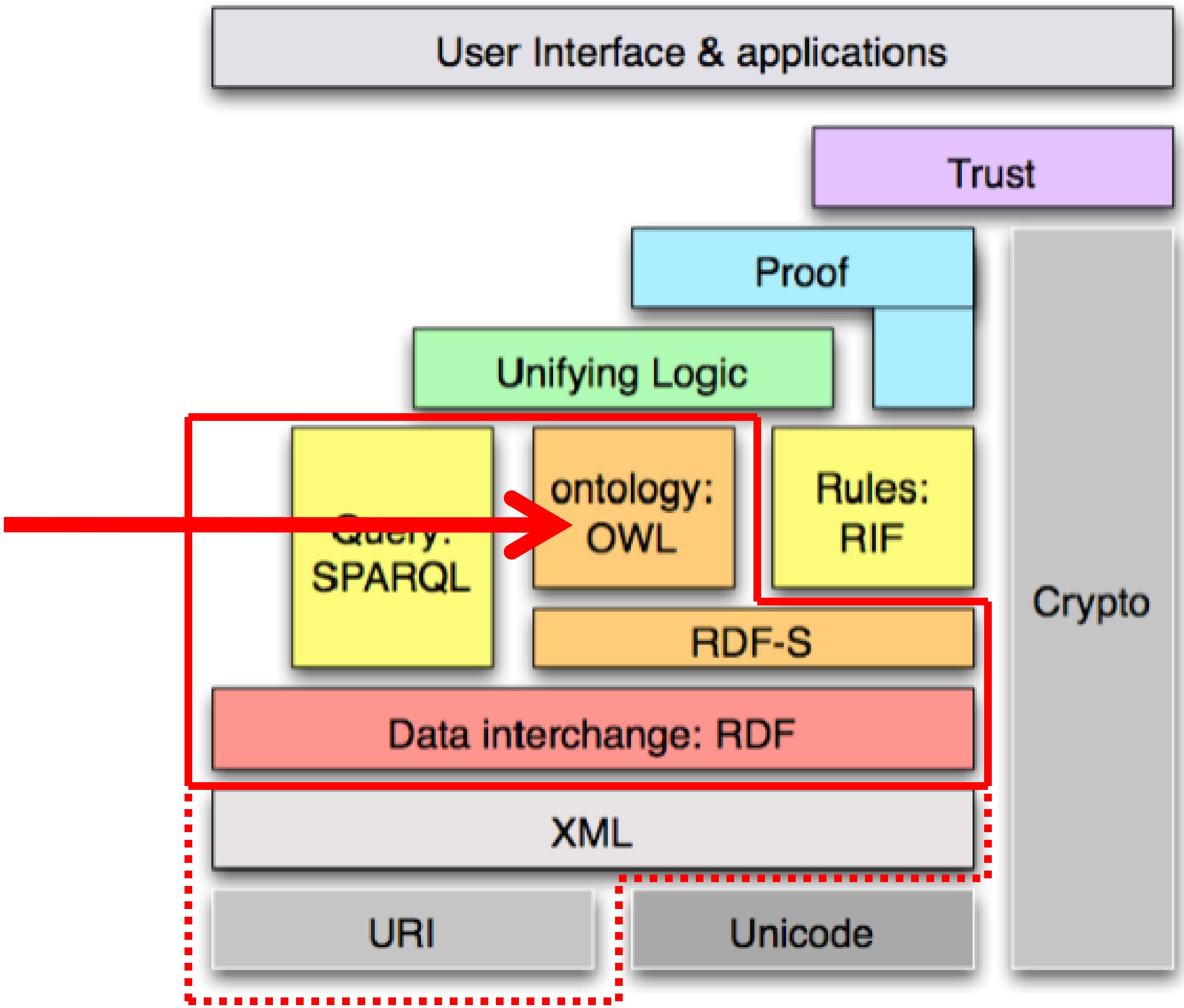
A Semantic Puzzle

From Horridge, Parsia, Sattler, From Justifications to Proofs for Entailments in OWL. In: Proceedings OWLED2009.
http://sunsite.informatik.rwth-aachen.de/Publications/CEUR-WS/Vol-529/

$$\mathsf{Person} \sqsubseteq \neg\mathsf{Movie}$$
$$\mathsf{RRated} \sqsubseteq \mathsf{CatMovie}$$
$$\mathsf{CatMovie} \sqsubseteq \mathsf{Movie}$$
$$\mathsf{RRated} \equiv (\exists \mathsf{hasScript}.\mathsf{ThrillerScript}) \sqcup (\forall \mathsf{hasViolenceLevel}.\mathsf{High})$$
$$\mathsf{Domain}(\mathsf{hasViolenceLevel},\ \mathsf{Movie})$$

Fig. 1. A justification for Person $\sqsubseteq \bot$

Today: Model-theoretic Semantics



Today’s Session: DL Semantics

1. Model-theoretic Semantics of SROIQ(D)
2. The Description Logic EL++
3. Class Project
4. Class Presentations

Model-theoretic Semantics

- **Recall:**

  How does one make a model-theoretic semantics?

  What – which mathematical entity – actually captures the "meaning"?

  How would we get at this here?

Model-theoretic Semantics

- There are two semantics for OWL.

1. Description Logic Semantics
   also: Direct Semantics; FOL Semantics
   Can be obtained by translation to FOL.
   Some global restrictions apply! (see next slide)

2. RDF-based Semantics (requires RDF/XML syntax: done later)
   No syntax restrictions apply.
   Extends the direct semantics with RDFS-reasoning features.

In the following, we will deal with the direct semantics only.

Direct Semantics

To obtain decidability, syntactic restrictions apply.

- **Type separation / punning**
- **No cycles in property chains.**
  (See global restrictions mentioned earlier.)
- **No transitive properties in cardinality restrictions.**
  (See global restrictions mentioned earlier.)

Decidability

- A problem is *decidable* if there exists an always terminating algorithm which determines, whether or not a solution exists.
- A problem is *semi-decidable* if there exists an algorithm which, in case a solution exists, finds this out in finite time.
- A problem is *undecidable* if if it not decidable.
- Note there exist problems which are semi-decidable and undecidable.

Decidability of DLs

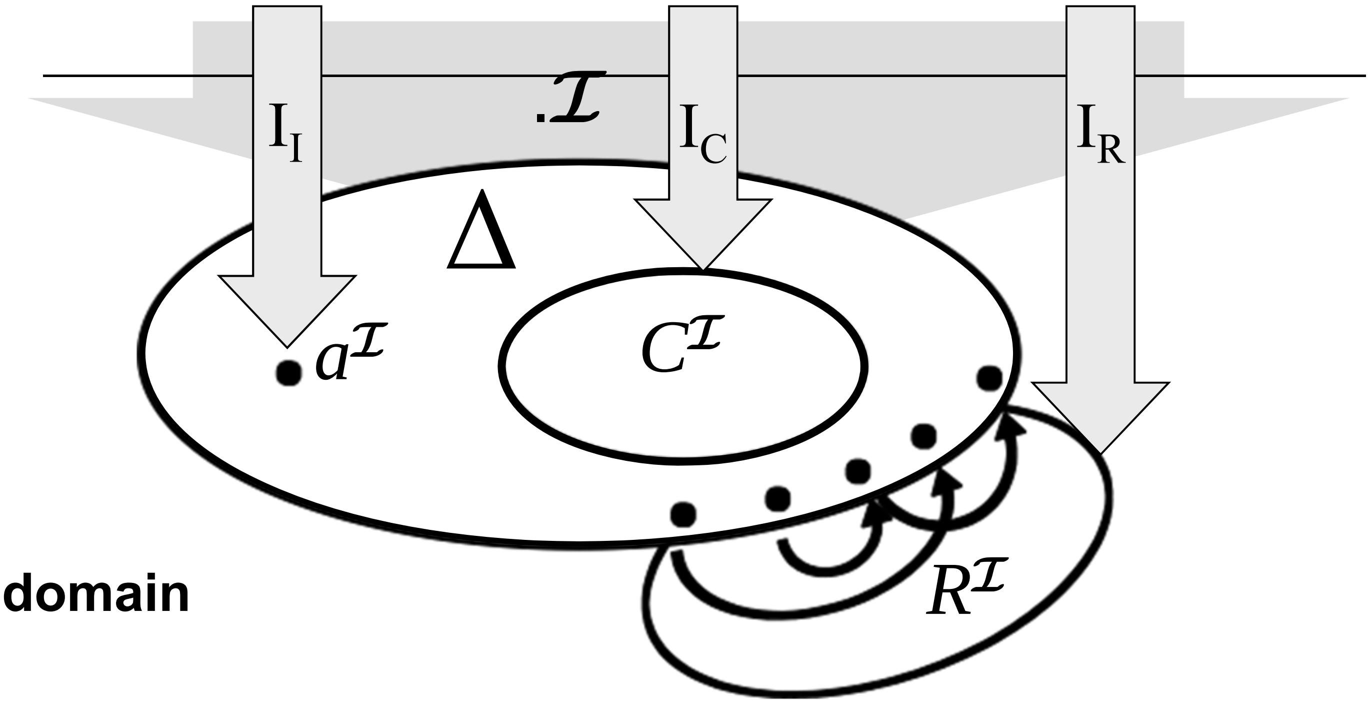
- A description logic is decidable if “entailment of axioms” is decidable.

- Most description logics are decidable.
  Decidability is one of the design criteria for “good” description logics.

Direct Semantics

- model-theoretic semantics
- starts with interpretations
- an interpretation $\mathcal{I}$ maps

  individual names, class names and property names...



...into a domain

Interpretation Example

If we consider, for example, the knowledge base consisting of the axioms

$$\texttt{Professor} \sqsubseteq \texttt{FacultyMember}$$
$$\texttt{Professor}(\texttt{rudiStuder})$$
$$\texttt{hasAffiliation}(\texttt{rudiStuder}, \texttt{aifb})$$

then we could set

$$\begin{aligned}
\Delta &= \{a, b, \text{Ian}\} \\
I_{\mathbf{I}}(\texttt{rudiStuder}) &= \text{Ian} \\
I_{\mathbf{I}}(\texttt{aifb}) &= b \\
I_{\mathbf{C}}(\texttt{Professor}) &= \{a\} \\
I_{\mathbf{C}}(\texttt{FacultyMember}) &= \{a, b\} \\
I_{\mathbf{R}}(\texttt{hasAffiliation}) &= \{(a, b), (b, \text{Ian})\}
\end{aligned}$$

Intuitively, these settings are nonsense, but they nevertheless determine a valid interpretation.

OWL Direct Semantics

- **mapping is extended to complex class expressions:**
  - $\top^I = \Delta^I$ $\bot^I = \emptyset$
  - $(C \sqcap D)^I = C^I \cap D^I$ $(C \sqcup D)^I = C^I \cup D^I$ $(\neg C)^I = \Delta^I \setminus C^I$
  - $(\forall R.C)^I = \{ x \mid \text{for all } (x,y) \in R^I \text{ we have } y \in C^I \}$
    $(\exists R.C)^I = \{ x \mid \text{there is } (x,y) \in R^I \text{ with } y \in C^I \}$
  - $(\geq nR.C)^I = \{ x \mid \#\{ y \mid (x,y) \in R^I \text{ and } y \in C^I \} \geq n \}$
  - $(\leq nR.C)^I = \{ x \mid \#\{ y \mid (x,y) \in R^I \text{ and } y \in C^I \} \leq n \}$
- **...and to role expressions:**
  - $U^I = \Delta^I \times \Delta^I$ $(R^-)^I = \{ (y,x) \mid (x,y) \in R^I \}$
- **...and to axioms:**
  - $C(a)$ holds, if $a^I \in C^I$ $R(a,b)$ holds, if $(a^I,b^I) \in R^I$
  - $C \sqsubseteq D$ holds, if $C^I \subseteq D^I$ $R \sqsubseteq S$ holds, if $R^I \subseteq S^I$
  - Disjoint(R,S) holds if $R^I \cap S^I = \emptyset$
  - $S_1 \circ S_2 \circ \ldots \circ S_n \sqsubseteq R$ holds if $S_1^I \circ S_2^I \circ \ldots \circ S_n^I \subseteq R^I$

OWL Direct Semantics

- what's below gives us a notion of *model*:

  An interpretation is a model of a set of axioms if all the axioms hold (are evaluated to true) in the interpretation.

- Notion of *logical consequence* obtained as usual.

- ...and to axioms:
  - C(a) holds, if $a^I \in C^I$ R(a,b) holds, if $(a^I,b^I) \in R^I$
  - $C \sqsubseteq D$ holds, if $C^I \subseteq D^I$ $R \sqsubseteq S$ holds, if $R^I \subseteq S^I$
  - Disjoint(R,S) holds if $R^I \cap S^I = \emptyset$
  - $S_1 \circ S_2 \circ \ldots \circ S_n \sqsubseteq R$ holds if $S_1^I \circ S_2^I \circ \ldots \circ S_n^I \subseteq R^I$

Not a model!

If we consider, for example, the knowledge base consisting of the axioms

$$\texttt{Professor} \sqsubseteq \texttt{FacultyMember}$$
$$\texttt{Professor}(\texttt{rudiStuder})$$
$$\texttt{hasAffiliation}(\texttt{rudiStuder}, \texttt{aifb})$$

then we could set

$$\begin{aligned}
\Delta &= \{a, b, \text{Ian}\} \\
\mathrm{I}_{\mathbf{I}}(\texttt{rudiStuder}) &= \text{Ian} \\
\mathrm{I}_{\mathbf{I}}(\texttt{aifb}) &= b \\
\mathrm{I}_{\mathbf{C}}(\texttt{Professor}) &= \{a\} \\
\mathrm{I}_{\mathbf{C}}(\texttt{FacultyMember}) &= \{a, b\} \\
\mathrm{I}_{\mathbf{R}}(\texttt{hasAffiliation}) &= \{(a, b), (b, \text{Ian})\}
\end{aligned}$$

Intuitively, these settings are nonsense, but they nevertheless determine a valid interpretation.

A model

$$\text{Professor} \sqsubseteq \text{FacultyMember}$$
$$\text{Professor}(\text{rudiStuder})$$
$$\text{hasAffiliation}(\text{rudiStuder}, \text{aifb})$$

$$\Delta = \{a, r, s\}$$
$$I_{\mathbf{I}}(\text{rudiStuder}) = r$$
$$I_{\mathbf{I}}(\text{aifb}) = a$$
$$I_{\mathbf{C}}(\text{Professor}) = \{r\}$$
$$I_{\mathbf{C}}(\text{FacultyMember}) = \{r, s\}$$
$$I_{\mathbf{R}}(\text{hasAffiliation}) = \{(r, a)\}$$

Models

$$\mathtt{Professor} \sqsubseteq \mathtt{FacultyMember}$$

$$\mathtt{Professor(rudiStuder)}$$

$$\mathtt{hasAffiliation(rudiStuder, aifb)}$$

	Model 1	Model 2	Model 3
Δ	$\{a, r, s\}$	$\{1, 2\}$	$\{\spadesuit\}$
$I_I(\mathtt{rudiStuder})$	r	1	$\spadesuit$
$I_I(\mathtt{aifb})$	a	2	$\spadesuit$
$I_C(\mathtt{Professor})$	$\{r\}$	$\{1\}$	$\{\spadesuit\}$
$I_C(\mathtt{FacultyMember})$	$\{a, r, s\}$	$\{1, 2\}$	$\{\spadesuit\}$
$I_R(\mathtt{hasAffiliation})$	$\{(r, a)\}$	$\{(1, 1), (1, 2)\}$	$\{(\spadesuit, \spadesuit)\}$

Is FacultyMember(aifb) a logical consequence?

Counterexample

Returning to our running example knowledge base, let us show formally that $\texttt{FacultyMember}(\texttt{aifb})$ is not a logical consequence. This can be done by giving a model M of the knowledge base where $\texttt{aifb}^M \notin \texttt{FacultyMember}^M$. The following determines such a model.

$$
\begin{aligned}
\Delta &= \{a, r\} \\
\mathrm{I}_{\mathrm{I}}(\texttt{rudiStuder}) &= r \\
\mathrm{I}_{\mathrm{I}}(\texttt{aifb}) &= a \\
\mathrm{I}_{\mathrm{C}}(\texttt{Professor}) &= \{r\} \\
\mathrm{I}_{\mathrm{C}}(\texttt{FacultyMember}) &= \{r\} \\
\mathrm{I}_{\mathrm{R}}(\texttt{hasAffiliation}) &= \{(r, a)\}
\end{aligned}
$$

OWL Direct Semantics via FOL

- but often OWL 2 DL is said to be a fragment of first-order predicate logic (FOL) [with equality]...
- yes, there is a translation of OWL 2 DL into FOL

$$\pi(C \sqsubseteq D) = (\forall x)(\pi_x(C) \to \pi_x(D))$$
$$\pi_x(A) = A(x)$$
$$\pi_x(\neg C) = \neg \pi_x(C)$$
$$\pi_x(C \sqcap D) = \pi_x(C) \wedge \pi_x(D)$$
$$\pi_x(C \sqcup D) = \pi_x(C) \vee \pi_x(D)$$
$$\pi_x(\forall R.C) = (\forall x_1)(R(x, x_1) \to \pi_{x_1}(C))$$
$$\pi_x(\exists R.C) = (\exists x_1)(R(x, x_1) \wedge \pi_{x_1}(C))$$
$$\pi_x(\geq nS.C) = (\exists x_1)\dots(\exists x_n)\left(\bigwedge_{i \neq j}(x_i \neq x_j) \wedge \bigwedge_i (S(x, x_i) \wedge \pi_{x_i}(C))\right)$$
$$\pi_x(\leq nS.C) = \neg(\exists x_1)\dots(\exists x_{n+1})\left(\bigwedge_{i \neq j}(x_i \neq x_j) \wedge \bigwedge_i (S(x, x_i) \wedge \pi_{x_i}(C))\right)$$
$$\pi_x(\{a\}) = (x = a)$$
$$\pi_x(\exists S.\mathrm{Self}) = S(x, x)$$

$$\pi(R_1 \sqsubseteq R_2) = (\forall x)(\forall y)(\pi_{x,y}(R_1) \to \pi_{x,y}(R_2))$$
$$\pi_{x,y}(S) = S(x, y)$$
$$\pi_{x,y}(R^-) = \pi_{y,x}(R)$$
$$\pi_{x,y}(R_1 \circ \dots \circ R_n) = (\exists x_1)\dots(\exists x_{n-1})\left(\pi_{x,x_1}(R_1) \wedge \bigwedge_{i=1}^{n-2} \pi_{x_i,x_{i+1}}(R_{i+1}) \wedge \pi_{x_{n-1},y}(R_n)\right)$$
$$\pi(\mathrm{Ref}(R)) = (\forall x)\pi_{x,x}(R)$$
$$\pi(\mathrm{Asy}(R)) = (\forall x)(\forall y)(\pi_{x,y}(R) \to \neg\pi_{y,x}(R)$$
$$\pi(\mathrm{Dis}(R_1, R_2)) = \neg(\exists x)(\exists y)(\pi_{x,y}(R_1) \wedge \pi_{x,y}(R_2))$$

- ...which (interpreted under FOL semantics) coincides with the definition just given.

Inconsistency and Satisfiability

- **A set of axioms (knowledge base) is satisfiable (or consistent) if it has a model.**
- **It is unsatisfiable (inconsistent) if it does not have a model.**

- **Inconsistency is often caused by modelling errors.**

$$\begin{aligned}
\mathtt{Unicorn}(\mathtt{beautyTheUnicorn}) \\
\mathtt{Unicorn} &\sqsubseteq \mathtt{Fictitious} \\
\mathtt{Unicorn} &\sqsubseteq \mathtt{Animal} \\
\mathtt{Fictitious} \sqcap \mathtt{Animal} &\sqsubseteq \bot
\end{aligned}$$

Inconsistency and Satisfiability

- **A knowledge base is incoherent if a named class is equivalent to ⊥.**
- **It usually also points to a modeling error.**

$$
\begin{aligned}
\mathtt{Unicorn} &\sqsubseteq \mathtt{Fictitious} \\
\mathtt{Unicorn} &\sqsubseteq \mathtt{Animal} \\
\mathtt{Fictitious} \sqcap \mathtt{Animal} &\sqsubseteq \bot
\end{aligned}
$$

Rationale behind OWL

- Open World Assumption
- Favourable trade-off between expressivity and scalability
- Integrates with RDFS
- Purely declarative semantics

Features:

- Fragment of first-order predicate logic (FOL)
- Decidable
- Known complexity classes (N2ExpTime for OWL 2 DL)
- Reasonably efficient for real KBs

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$$\mathsf{Person} \sqsubseteq \neg\mathsf{Movie}$$

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Fig. 1. A justification for Person $\sqsubseteq \bot$

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OWL EL

- The OWL 2 spec describes three profiles (fragments, sublanguages) which have polynomial complexity.
  - OWL EL (the description logic EL++)
    we will talk about this next
  - OWL QL (the description logic DL Lite$_R$)
    forthcoming class presentation
  - OWL RL (the description logic DLP)
    skipped
    - inspired by intersecting OWL with Datalog
    - implemented e.g. in Oracle 11g

EL++

- *Pushing the EL Envelope*. Franz Baader, Sebastian Brandt, and Carsten Lutz. In Proc. of the 19th Joint Int. Conf. on Artificial Intelligence (IJCAI 2005), 2005
  - this introduces EL++

- *Pushing the EL Envelope Further*. Franz Baader, Sebastian Brandt, and Carsten Lutz. In Proc. of the Washington DC workshop on OWL: Experiences and Directions (OWLED08DC), 2008
  - this extends EL++. If people talk about EL++ better check if the extended version is meant.

EL++

- **EL**
  - **existential quantification $\exists$**
  - **conjunction $\sqcap$**
  - **top concept $\top$**
  - **i.e. it's a fragment/sublanguage of ALC**
- **EL+**
  - **bottom concept $\bot$** **(this allows e.g. disjoint classes)**
  - **role chains $R \circ S \sqsubseteq T$**
  - **datatypes**
- **EL++**
  - **nominals with one individual {o}**
- **EL++ extended**
  - **reflexive roles**
  - **range of roles** **(domain is already in EL)**

note: a global syntactic restriction applies to guarantee polynomiality

EL++ (ext.) global restriction

- **IF**
  - $R_1 \circ \ldots \circ R_n \sqsubseteq S_1$
  - $S_1 \sqsubseteq \ldots \sqsubseteq S_n$
  - $\text{range}(S_n) \sqsubseteq C$
- **THEN**
  - there are $R_{n+1},\ldots,R_m$ with
  - $R_n \sqsubseteq R_{n+1} \sqsubseteq \ldots \sqsubseteq R_m$ and
  - $\text{range}(R_m) \sqsubseteq C$

SNOMED

- Work on EL++ initiated a research branch into polynomial description logics.
- Breakthrough was the classification of the SNOMED commercial ontology.
  – http://www.ihtsdo.org/snomed-ct/

EL++

- **Most well-known reasoner: CEL http://lat.inf.tu-dresden.de/systems/cel/**
  - **performs *classification* only:
    computation of the class hierarchy of all named classes**
- **Pellet also has a specialized algorithm implemented**
- **It's currently still unclear how to reason efficiently with nominals (and thus with ABoxes).**

Other polynomial OWL profiles

- **See http://www.w3.org/TR/owl2-profiles/**

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Class Project

- Use the classes and properties from your ontology (if necessary, add some new ones).
- Use them as class names and role names, and write down (in DL notation) a number of SROIQ axioms which make sense in the context of your project ontology.
- Make sure you use each of the following constructs at least once:
  - $\sqcap$, $\sqcup$, $\neg$, $\exists$, $\forall$
  - a nominal
  - an inverse property
  - a qualified cardinality constraint
  - three of the property characteristics

Class Project

- Send me by Sunday 21st of February:
  - Current version of your ontology in Turtle syntax.
  - The DL axioms.
    - Either on paper, handwritten (e.g. via Tonya Davis for me)
    - Or as a pdf (e.g. generated from LaTex).
    - Or via Protege (in one of the OWL 2 serializations). (We haven't talked about OWL 2 syntax yet, so this is really optional.)
  - Each DL axioms accompanied with a natural language sentence which captures its meaning.

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Class presentations – scheduled

- RDFa – embedding RDF in HTML (W3C standard)
  Pavan, Thursday 28th of January

- Scalable Distributed Reasoning using MapReduce (Urbani, Kotoulas, Oren, van Harmelen, ISWC2009)
  Wenbo, Thursday 28th of January

All remaining presentations will be in the last week

- **Semantic MediaWiki, Vinh, to be scheduled**
- **Linked Open Data, Ashutosh, to be scheduled**
- **FOAF, Hemant, to be scheduled**
- **Virtuoso, Pramod, to be scheduled**
- **Prateek, Conjunctive Queries for OWL**
- **Raghava, DL-Lite**

Class Planning

Thursday 4th of February: OWL Part 1

Tuesday 9th of February: Campus Closed

Thursday 11th of February: OWL Part 2

Tuesday 23rd of February: Exercise Session

Thursday 25th of February: OWL Part 3

Week from March 8th: Class Presentations

Friday March 12th: most exams

Estimated breakdown of sessions:

Intro + XML: 2

RDF: 3.3

OWL: 4

SPARQL: 1

Class Project Session: 1

Class Presentations: 3

Exercise sessions: 2.7