



# OWL 2 Rules (Part 1)

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# Outline Part 1

- **The Early Days of KR: Rule-Based Formalisms**
- **OWL 2 DL – the new DL-based Web Ontology Language**
- **Semantics of OWL DL**
- **Tractable Fragments**

# The Early Days of KR: Rule-Based Formalisms

- rules provide a natural way of modelling “if-then“ knowledge
- general form of a (Horn) rule:

Body  $\rightarrow$  Head

- body: (possibly empty) conjunction of atoms, head: at most one atom
- Examples:

$\text{married}(x,y) \wedge \text{Woman}(x) \rightarrow \text{Man}(y)$

$\text{Man}(x) \wedge \text{Woman}(x) \rightarrow$

$\rightarrow \text{married}(\text{pascal}, \text{anne})$

# The Early Days of KR: Rule-Based Formalisms

- rules provide a natural way of modelling “if-then“ knowledge
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- Examples:

$$\forall x \forall y (\text{married}(x,y) \wedge \text{Woman}(x) \rightarrow \text{Man}(y))$$
$$\forall x (\text{Man}(x) \wedge \text{Woman}(x) \rightarrow \text{false})$$
$$\text{true} \rightarrow \text{married}(\text{pascal}, \text{anne})$$

# On the Semantics of Rules

- syntactically, rules are just FOL formulae
- hence they can be interpreted under FOL standard semantics
- other (non-monotonic) interpretations are possible:
  - well-founded semantics
  - stable model semantics
  - answer set semantics
- in the case of Horn rules, they all coincide (differences if negation of atoms is allowed)
- in this tutorial, we strictly adhere to FOL (=open-world) semantics

# What We Cannot Say with Rules

- with rules, one cannot require the existence of individuals with certain properties except by explicitly naming them
- i.e. we can express that there are two persons that are married by giving them names (say, person1 and person2):

$\text{true} \rightarrow \text{married}(\text{person1}, \text{person2})$

- but we cannot express something like:  
“every husband is married to somebody“

**wrong:**

$\text{husband}(x) \rightarrow \text{married}(x, \text{person})$

**That's where  
OWL comes in!**

# What OWL Talks About (Semantics)

- both OWL 1 DL and OWL 2 DL are based on description logics
- here, we will treat OWL from the “description logic viewpoint“:
  - we use DL syntax
  - we won't talk about datatypes and non-semantic features of OWL
- OWL (DL) ontologies talk about worlds that contain

individuals

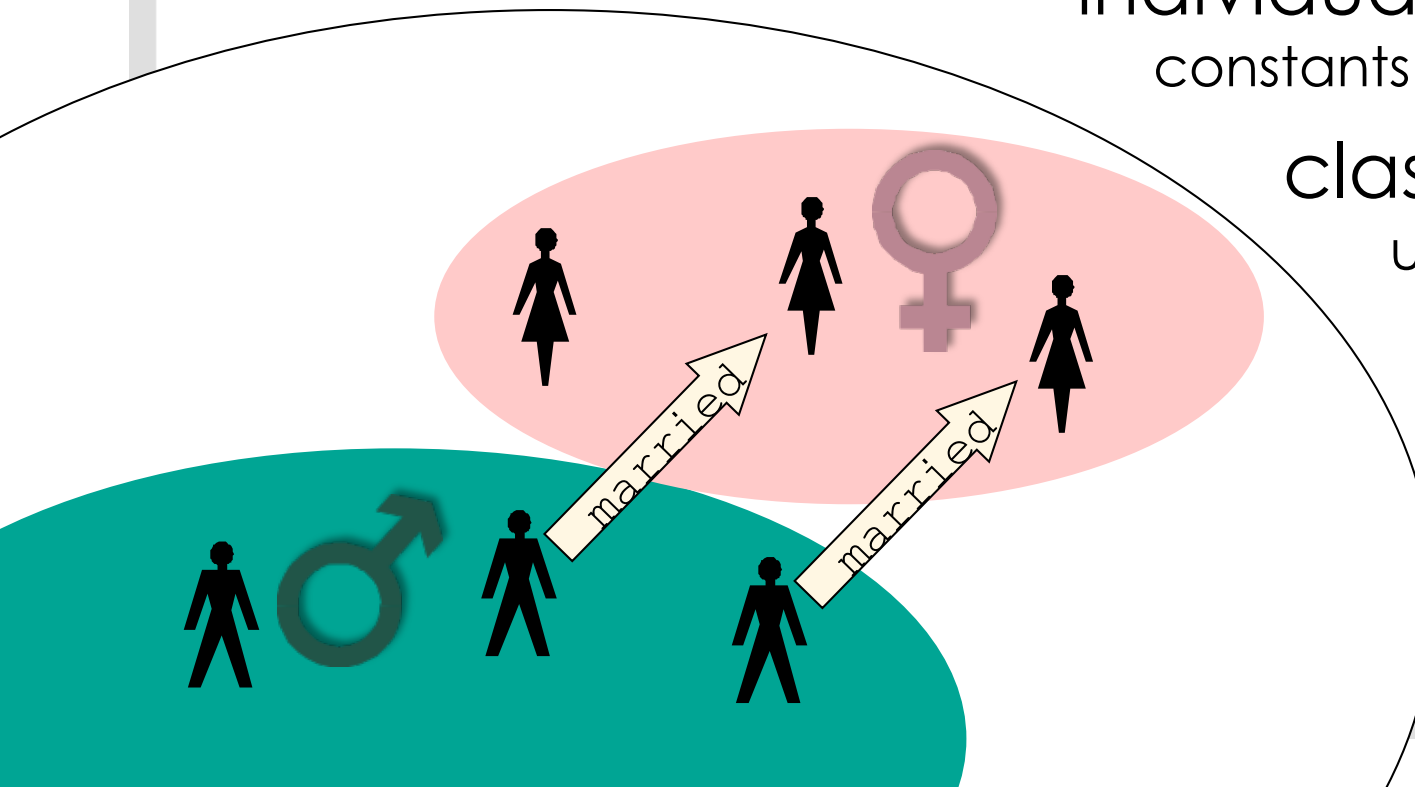
constants: pascal, anne

classes / concepts

unary predicates:  
male(\_), female(\_)

properties / roles

binary predicates:  
married(\_,\_)



# Assertional Knowledge

- asserts information about concrete named individuals

- class membership: Male(pascal)

```
<Male rdf:about="pascal"/>
```

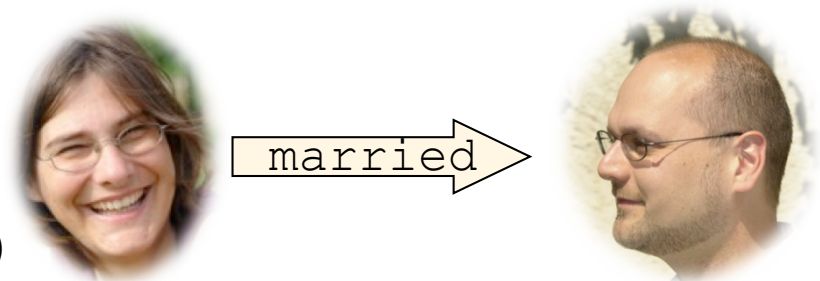
rule version: → Male(pascal)



- property membership: married(anne,pascal)

```
<rdf:Description rdf:about="anne">  
  <married rdf:resource="pascal"/>  
</rdf:Description>
```

rule version: → married (anne,pascal)



## That's all what can be said with RDF!



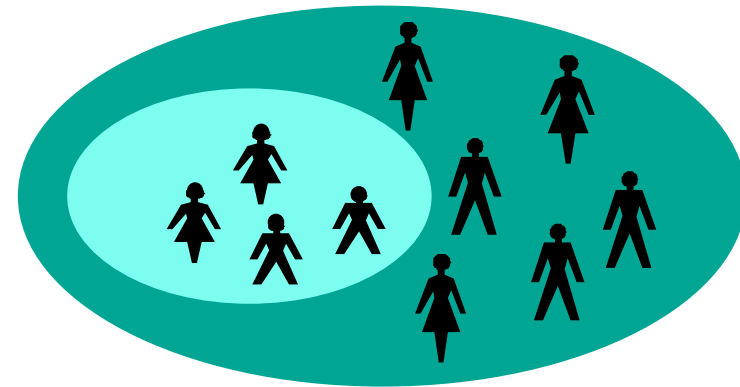
# Terminological Knowledge – Subclasses and Subproperties

- information about how classes and properties relate in general

- subclass:  $\text{Child} \sqsubseteq \text{Person}$

```
<owl:Class rdf:about="Child">
  <rdfs:subClassOf rdf:resource="Person"/>
</owl:Class>
```

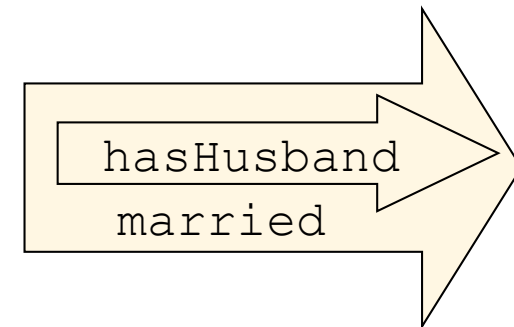
rule version:  $\text{Child}(x) \rightarrow \text{Person}(x)$



- subproperty:  $\text{hasHusband} \sqsubseteq \text{married}$

```
<owl:ObjectProperty rdf:about="hasHusband">
  <rdfs:subPropertyOf rdf:resource="married"/>
</owl:ObjectProperty>
```

rule version:  $\text{hasHusband}(x,y) \rightarrow \text{married}(x,y)$

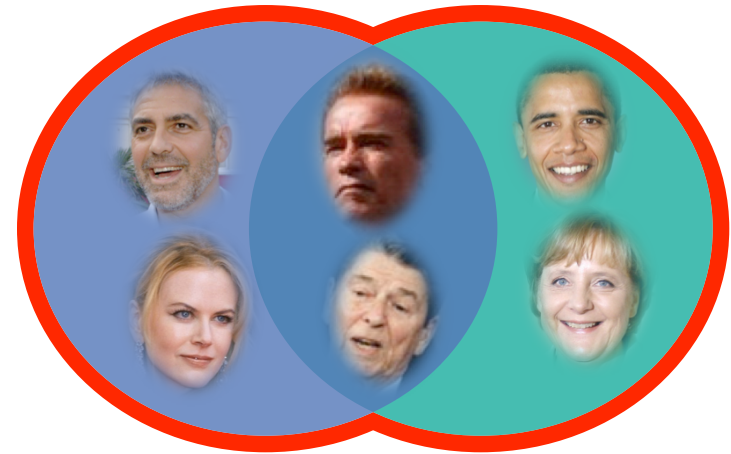


# Class Constructors

- build new classes from class, property and individual names

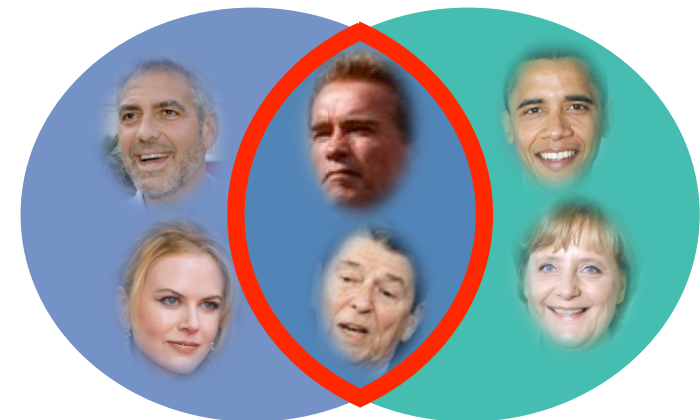
- union: Actor  $\sqcup$  Politician

```
<owl:unionOf rdf:parseType="Collection">  
  <owl:Class rdf:about="Actor"/>  
  <owl:Class rdf:about="Politician"/>  
</owl:unionOf>
```



- intersection: Actor  $\sqcap$  Politician

```
<owl:intersectionOf rdf:parseType="Collection">  
  <owl:Class rdf:about="Actor"/>  
  <owl:Class rdf:about="Politician"/>  
</owl:intersectionOf>
```



# Class Constructors

- build new classes from class, property and individual names

- complement:  $\neg$ Politician

```
<owl:complementOf  
  rdf:resource="Politician">
```



- closed classes: {anne,merula,pascal}

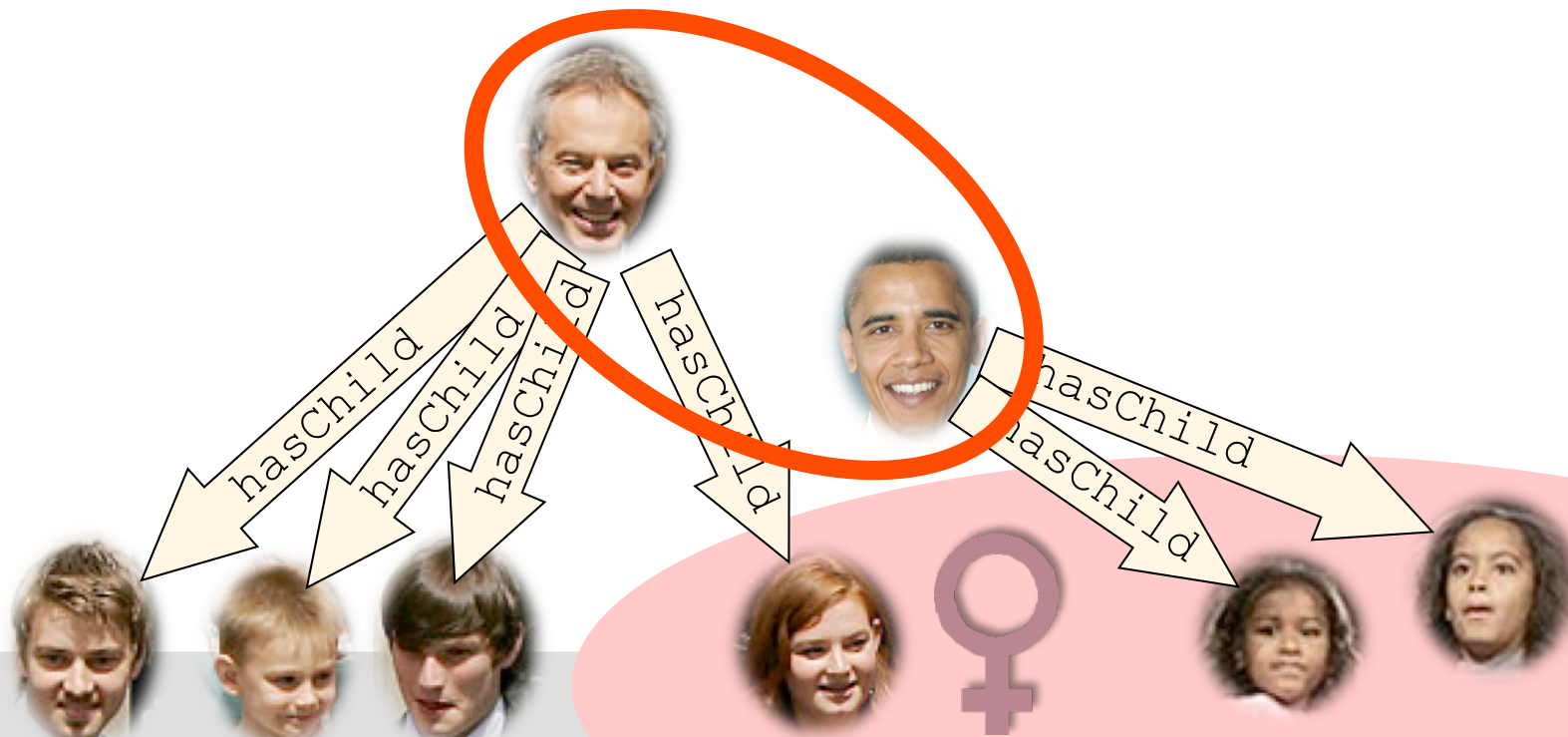
```
<owl:oneOf rdf:parseType="Collection">  
  <rdf:Description rdf:about="anne"/>  
  <rdf:Description rdf:about="merula"/>  
  <rdf:Description rdf:about="pascal"/>  
</owl:oneOf>
```



# Class Constructors

- build new classes from class, property and individual names
  - existential quantification:  $\exists$ hasChild.Female

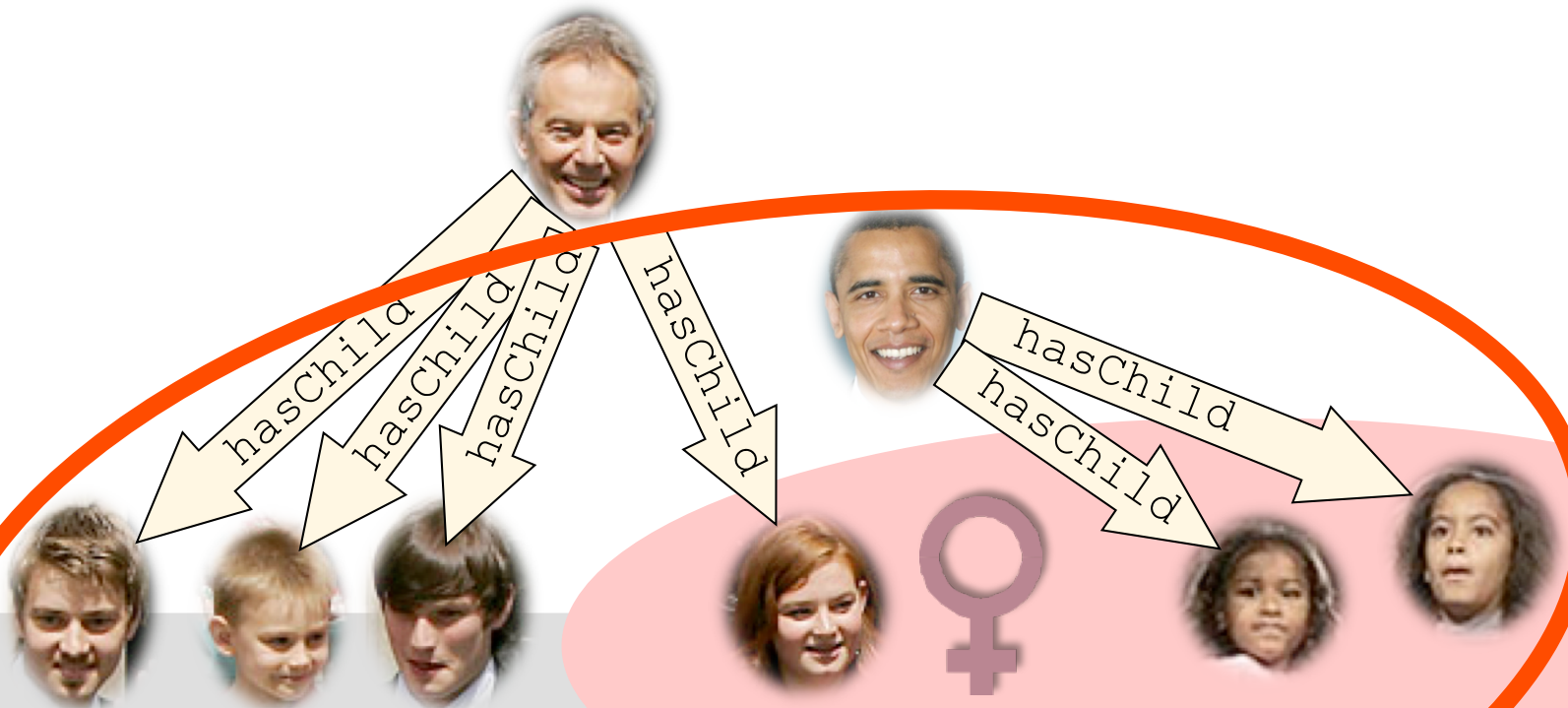
```
<owl:Restriction>  
  <owl:onProperty rdf:resource="hasChild"/>  
  <owl:someValuesFrom rdf:resource="Female"/>  
</owl:Restriction>
```



# Class Constructors

- build new classes from class, property and individual names
  - universal quantification:  $\forall \text{hasChild.Female}$

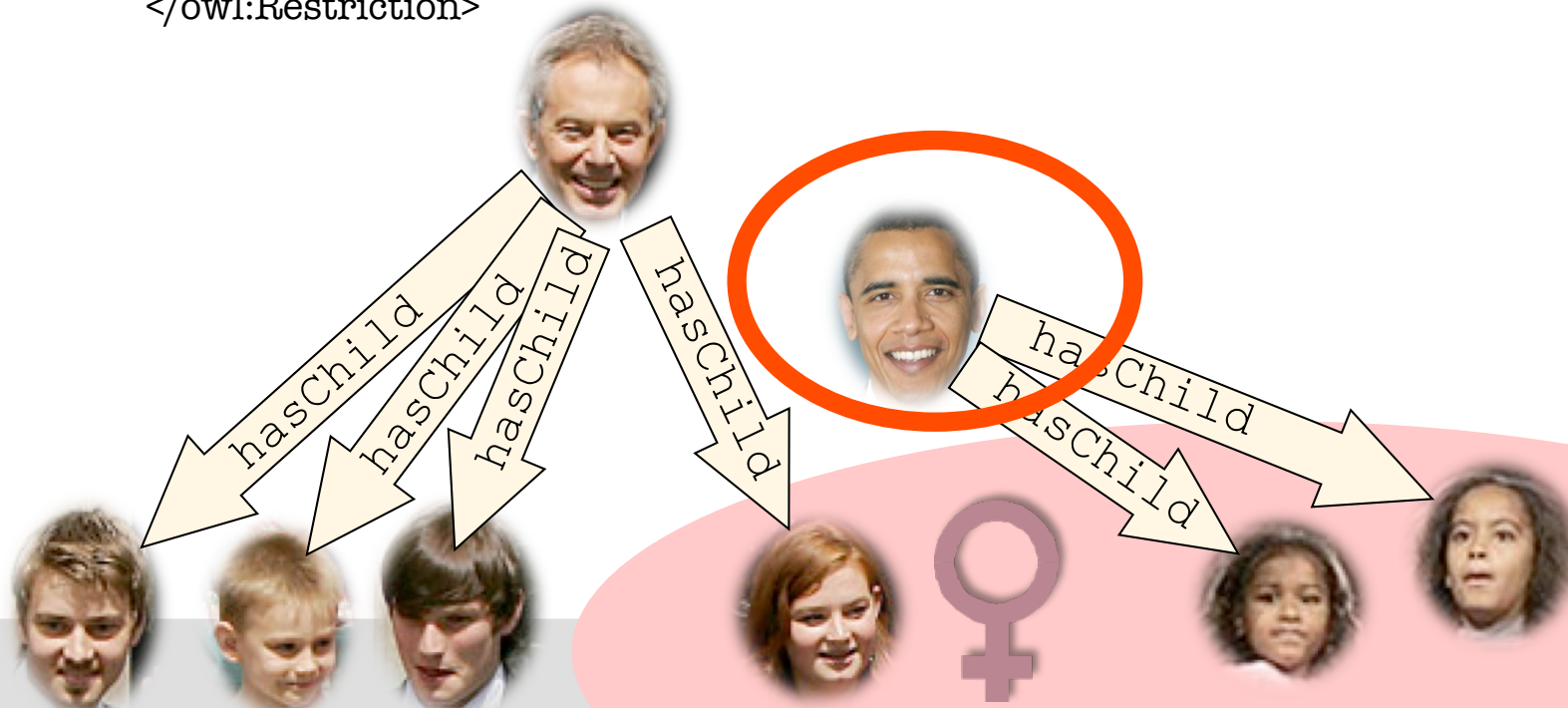
```
<owl:Restriction>  
  <owl:onProperty rdf:resource="hasChild"/>  
  <owl:allValuesFrom rdf:resource="Female"/>  
</owl:Restriction>
```



# Class Constructors

- build new classes from class, property and individual names
  - cardinality restriction:  $\geq 2$ hasChild.Female

```
<owl:Restriction>  
  <owl:minQualifiedCardinality rdf:datatype="&xsd;nonNegativeInteger">  
    2 </owl:minQualifiedCardinality>  
  <owl:onProperty rdf:about="hasChild"/>  
  <owl:onClass rdf:about="Female"/>  
</owl:Restriction>
```

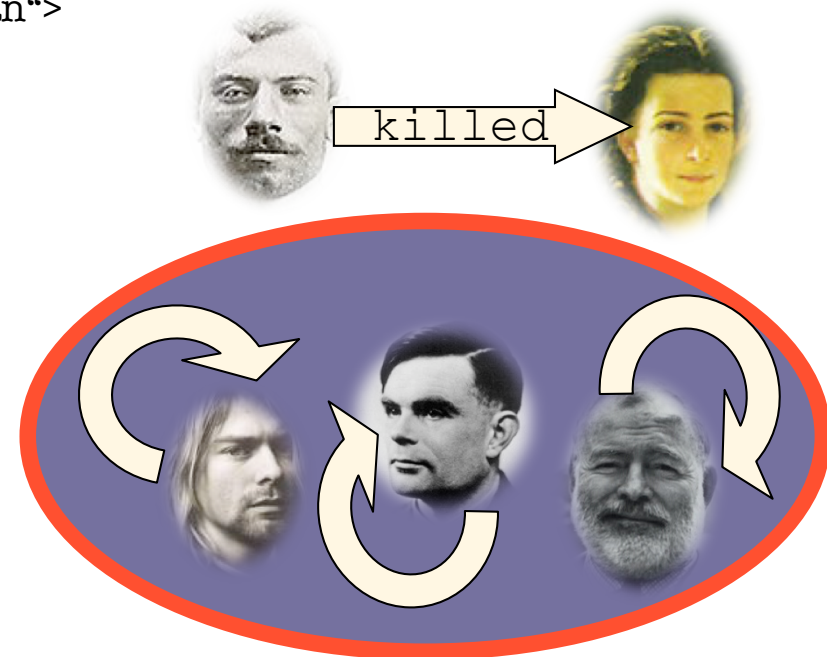




# Class Constructors

- build new classes from class, property and individual names
  - Self-restriction:  $\exists$ killed.Self

```
<owl:Restriction>  
  <owl:onProperty rdf:resource="killed"/>  
  <owl:hasSelf rdf:datatype="&xsd:boolean">  
    true  
  </owl:hasSelf>  
</owl:Restriction>
```



# Special Classes and Properties

- special classes

- top class:  $\top$

- ...class containing all individuals of the domain

- `owl:Thing`

- bottom class:  $\perp$

- ...“empty“ class containing no individuals

- `owl:Nothing`

- universal property:  $U$

- ...property linking every individual to every individual

- `owl:topObjectProperty`

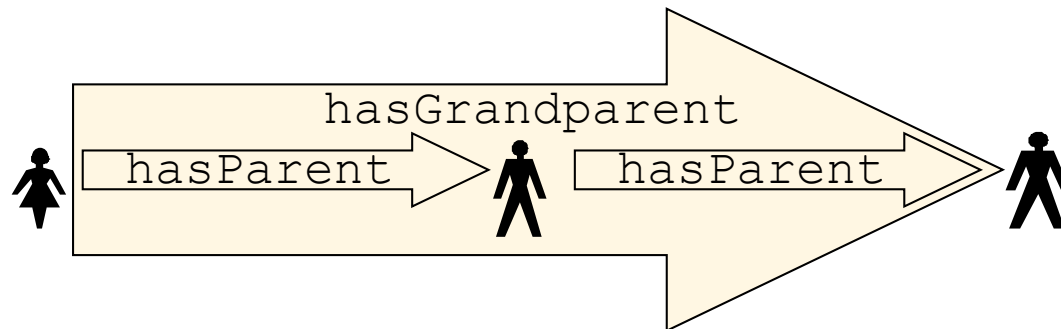


# Property Chain Axioms

- allow to infer the existence of a property from a chain of properties:

- $\text{hasParent} \circ \text{hasParent} \sqsubseteq \text{hasGrandparent}$

rule version:  $\text{hasParent}(x,y) \wedge \text{hasParent}(y,z) \rightarrow \text{hasGrandparent}(x,z)$



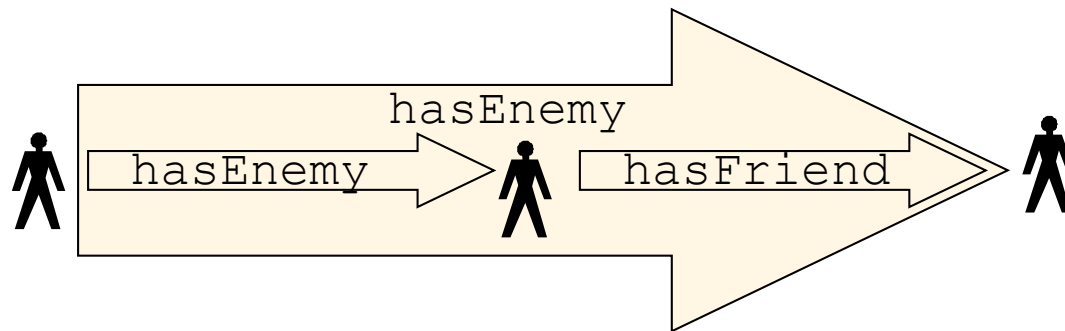
```
<rdf:Description rdf:about="hasGrandparent">  
  <owl:propertyChainAxiom rdf:parseType="Collection">  
    <owl:ObjectProperty rdf:about="hasParent"/>  
    <owl:ObjectProperty rdf:about="hasParent"/>  
  </owl:propertyChainAxiom>  
</rdf:Description>
```

# Property Chain Axioms

- allow to infer the existence of a property from a chain of properties:

- $\text{hasEnemy} \circ \text{hasFriend} \sqsubseteq \text{hasEnemy}$

rule version:  $\text{hasEnemy}(x,y) \wedge \text{hasFriend}(y,z) \rightarrow \text{hasEnemy}(x,z)$



```
<rdf:Description rdf:about="hasEnemy">
  <owl:propertyChainAxiom rdf:parseType="Collection">
    <owl:ObjectProperty rdf:about="hasEnemy"/>
    <owl:ObjectProperty rdf:about="hasFriend"/>
  </owl:propertyChainAxiom>
</rdf:Description>
```

# Property Chain Axioms: Caution! (1/2)

- arbitrary property chain axioms lead to undecidability
- restriction: set of property chain axioms has to be *regular*
  - there must be a strict linear order  $<$  on the properties
  - every property chain axiom has to have one of the following forms:
 

$R \circ R \sqsubseteq R$	$S^- \sqsubseteq R$	$S_1 \circ S_2 \circ \dots \circ S_n \sqsubseteq R$
$R \circ S_1 \circ S_2 \circ \dots \circ S_n \sqsubseteq R$		$S_1 \circ S_2 \circ \dots \circ S_n \circ R \sqsubseteq R$
  - thereby,  $S_i < R$  for all  $i = 1, 2, \dots, n$ .
- Example 1:  $R \circ S \sqsubseteq R$        $S \circ S \sqsubseteq S$        $R \circ S \circ R \sqsubseteq T$   
 → regular with order  $S < R < T$
- Example 2:  $R \circ T \circ S \sqsubseteq T$   
 → not regular because form not admissible
- Example 3:  $R \circ S \sqsubseteq S$        $S \circ R \sqsubseteq R$   
 → not regular because no adequate order exists

# Property Chain Axioms: Caution! (2/2)

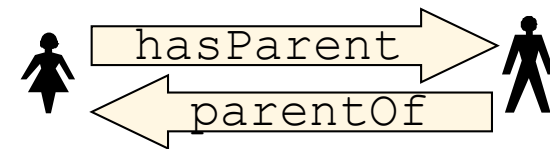
- combining property chain axioms and cardinality constraints may lead to undecidability
- restriction: use only *simple* properties in cardinality expressions (i.e. those which cannot be – directly or indirectly – inferred from property chains)
- technically:
  - for any property chain axiom  $S_1 \circ S_2 \circ \dots \circ S_n \sqsubseteq R$  with  $n > 1$ ,  $R$  is non-simple
  - for any subproperty axiom  $S \sqsubseteq R$  with  $S$  non-simple,  $R$  is non-simple
  - all other properties are simple
- Example:  $Q \circ P \sqsubseteq R$      $R \circ P \sqsubseteq R$      $R \sqsubseteq S$      $P \sqsubseteq R$      $Q \sqsubseteq S$   
non-simple:  $R, S$                   simple:  $P, Q$

# Property Characteristics

- a property can be
  - the inverse of another property:  $\text{hasParent} \equiv \text{parentOf}^{-1}$

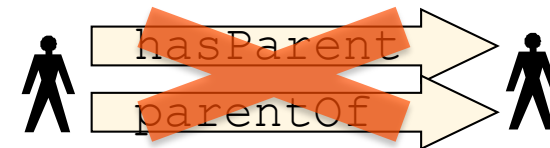
rule version:

$$\begin{aligned}\text{hasParent}(x,y) &\rightarrow \text{parentOf}(y,x) \\ \text{parentOf}(x,y) &\rightarrow \text{hasParent}(y,x)\end{aligned}$$



- disjoint with another property:  $\text{Disj}(\text{hasParent}, \text{parentOf})$
- rule version:

$$\text{hasParent}(x,y), \text{parentOf}(x,y) \rightarrow \text{false}$$

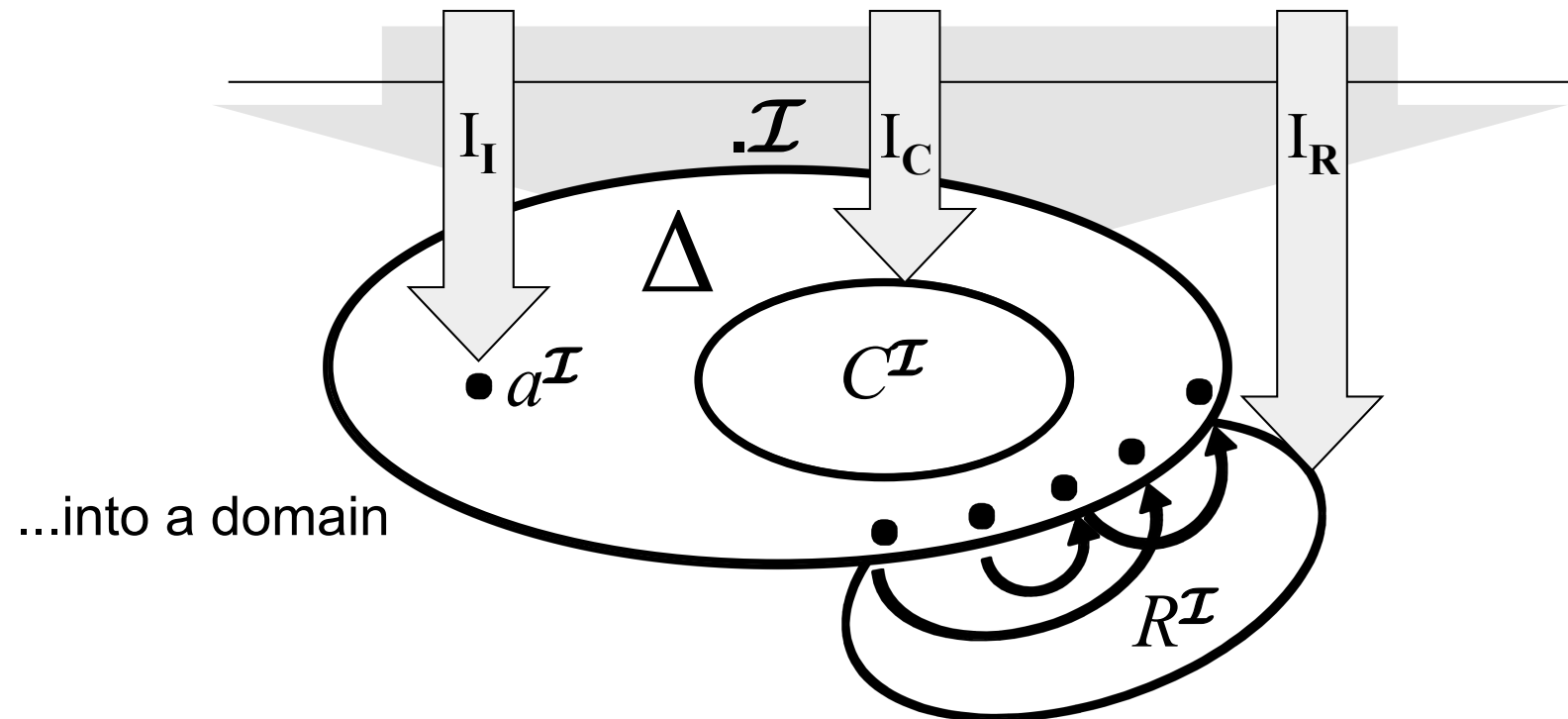


- other property characteristics that can be expressed:  
(inverse) functionality, transitivity, symmetry, asymmetry, reflexivity, irreflexivity

# OWL 2 DL – Semantics

- model-theoretic semantics
- starts with interpretations
- an interpretation maps

individual names, class names and property names...



- mapping is extended to complex class expressions:

- $\top^I = \Delta^I$                        $\perp^I = \emptyset$
- $(C \sqcap D)^I = C^I \cap D^I$              $(C \sqcup D)^I = C^I \cup D^I$              $(\neg C)^I = \Delta^I \setminus C^I$
- $\forall R.C = \{ x \mid \forall (x,y) \in R^I \rightarrow y \in C^I \}$      $\exists R.C = \{ x \mid \exists (x,y) \in R^I \wedge y \in C^I \}$
- $\geq n R.C = \{ x \mid \#\{ y \mid (x,y) \in R^I \wedge y \in C^I \} \geq n \}$
- $\leq n R.C = \{ x \mid \#\{ y \mid (x,y) \in R^I \wedge y \in C^I \} \leq n \}$

- ...and to role expressions:

- $U^I = \Delta^I \times \Delta^I$                        $(R^-)^I = \{ (y,x) \mid (x,y) \in R^I \}$

- ...and to axioms:

- $C(a)$  holds, if  $a^I \in C^I$                        $R(a,b)$  holds, if  $(a^I,b^I) \in R^I$
- $C \sqsubseteq D$  holds, if  $C^I \subseteq D^I$                        $R \sqsubseteq S$  holds, if  $R^I \subseteq S^I$
- $\text{Disj}(R,S)$  holds if  $R^I \cap S^I = \emptyset$
- $S_1 \circ S_2 \circ \dots \circ S_n \sqsubseteq R$  holds if  $S_1^I \circ S_2^I \circ \dots \circ S_n^I \subseteq R^I$

# OWL 2 DL – Alternative Semantics

- but often OWL 2 DL is said to be a fragment of FOL...
- yes, there is a translation of OWL 2 DL into FOL

$$\begin{aligned}
 \pi(C \sqsubseteq D) &= (\forall x)(\pi_x(C) \rightarrow \pi_x(D)) \\
 \pi_x(A) &= A(x) \\
 \pi_x(\neg C) &= \neg \pi_x(C) \\
 \pi_x(C \sqcap D) &= \pi_x(C) \wedge \pi_x(D) \\
 \pi_x(C \sqcup D) &= \pi_x(C) \vee \pi_x(D) \\
 \pi_x(\forall R.C) &= (\forall x_1)(R(x, x_1) \rightarrow \pi_{x_1}(C)) \\
 \pi_x(\exists R.C) &= (\exists x_1)(R(x, x_1) \wedge \pi_{x_1}(C)) \\
 \pi_x(\geq n S.C) &= (\exists x_1) \dots (\exists x_n) \left( \bigwedge_{i \neq j} (x_i \neq x_j) \wedge \bigwedge_i (S(x, x_i) \wedge \pi_{x_i}(C)) \right) \\
 \pi_x(\leq n S.C) &= \neg (\exists x_1) \dots (\exists x_{n+1}) \left( \bigwedge_{i \neq j} (x_i \neq x_j) \wedge \bigwedge_i (S(x, x_i) \wedge \pi_{x_i}(C)) \right) \\
 \pi_x(\{a\}) &= (x = a) \\
 \pi_x(\exists S.Self) &= S(x, x) \\
 \pi(R_1 \sqsubseteq R_2) &= (\forall x)(\forall y)(\pi_{x,y}(R_1) \rightarrow \pi_{x,y}(R_2)) \\
 \pi_{x,y}(S) &= S(x, y) \\
 \pi_{x,y}(R^-) &= \pi_{y,x}(R) \\
 \pi_{x,y}(R_1 \circ \dots \circ R_n) &= (\exists x_1) \dots (\exists x_{n-1}) \\
 &\quad \left( \pi_{x,x_1}(R_1) \wedge \bigwedge_{i=1}^{n-2} \pi_{x_i,x_{i+1}}(R_{i+1}) \wedge \pi_{x_{n-1},y}(R_n) \right) \\
 \pi(\text{Ref}(R)) &= (\forall x)\pi_{x,x}(R) \\
 \pi(\text{Asy}(R)) &= (\forall x)(\forall y)(\pi_{x,y}(R) \rightarrow \neg \pi_{y,x}(R)) \\
 \pi(\text{Dis}(R_1, R_2)) &= \neg (\exists x)(\exists y)(\pi_{x,y}(R_1) \wedge \pi_{x,y}(R_2))
 \end{aligned}$$

- ...which (interpreted under FOL semantics) coincides with the definition just given.



# OWL 2 Profiles

- OWL 2 DL is very expressive (although decidable)
  - tool support for full OWL 2 DL difficult to achieve
- complexity for standard reasoning tasks:  $N^2ExpTime$ 
  - scalability cannot be guaranteed
- idea: identify subsets of OWL 2 DL which are
  - still sufficiently expressive
  - of lower complexity (preferably in PTime)
  - computationally easier to handle
- OWL 2 Profiles:
  - OWL EL
  - OWL RL
  - OWL QL

# OWL 2 EL

- allowed:
  - subclass axioms with intersection, existential quantification, top, bottom
    - closed classes must have only one member
  - property chain axioms, range restrictions (under certain conditions)
- disallowed:
  - negation, disjunction, arbitrary universal quantification, role inverses

$\sqcap \exists T \perp \sqsubseteq$     $\sqsubseteq$     $\sqcap \exists T \perp$

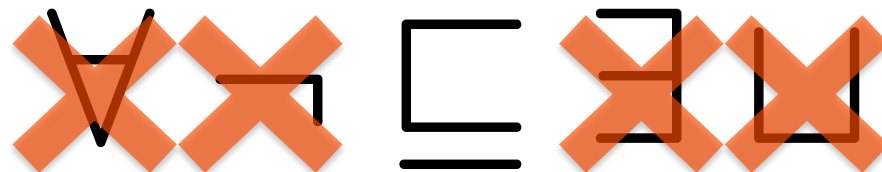
- Reasoning is PTime complete
- Examples:  $\exists \text{has.Sorrow} \sqsubseteq \exists \text{has.Liqueur}$     $\top \sqsubseteq \exists \text{hasParent.Person}$   
 $\exists \text{married.} \top \sqcap \text{CatholicPriest} \sqsubseteq \perp$     $\text{German} \sqsubseteq \exists \text{knows.}\{\text{angela}\}$   
 $\text{hasParent} \circ \text{hasParent} \sqsubseteq \text{hasGrandparent}$

# OWL 2 RL

- motivated by the question: what fraction of OWL 2 DL can be expressed by rules (with equality)?
- examples:
  - $\exists \text{parentOf}.\exists \text{parentOf}.\top \sqsubseteq \text{Grandfather}$   
rule version:  $\text{parentOf}(x,y) \wedge \text{parentOf}(y,z) \rightarrow \text{Grandfather}(x)$
  - $\text{Orphan} \sqsubseteq \forall \text{hasParent}.\text{Dead}$   
rule version:  $\text{Orphan}(x) \wedge \text{hasParent}(x,y) \rightarrow \text{Dead}(y)$
  - $\text{Monogamous} \sqsubseteq \leq 1 \text{married}.\text{Alive}$   
rule version:  
 $\text{Monogamous}(x) \wedge \text{married}(x,y) \wedge \text{Alive}(y) \wedge \text{married}(x,z) \wedge \text{Alive}(z) \rightarrow y=z$
  - $\text{childOf} \circ \text{childOf} \sqsubseteq \text{grandchildOf}$   
rule version:  $\text{childOf}(x,y) \wedge \text{childOf}(y,z) \rightarrow \text{grandchildOf}(x,z)$
  - $\text{Disj}(\text{childOf}, \text{parentOf})$   
rule version:  $\text{childOf}(x,y) \wedge \text{parentOf}(x,y) \rightarrow$

# OWL 2 RL

- syntactic characterization:
  - essentially, all axiom types are allowed
  - disallow certain constructors on lhs and rhs of subclass statements



- cardinality restrictions: only on rhs and only  $\leq 1$  and  $\leq 0$  allowed
  - closed classes: only with one member
- Reasoning is PTime complete
- Example Ontology: SWRC

# OWL 2 QL

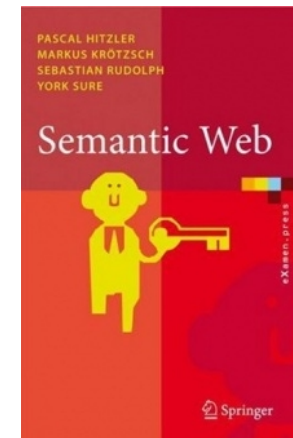
- motivated by the question: what fraction of OWL 2 DL can be captured by standard database technology?
- formally: query answering LOGSPACE w.r.t. data (via translation into SQL)
- allowed:
  - subproperties, domain, range
  - subclass statements with
    - left hand side: class name or expression of type  $\exists r.T$
    - right hand side: intersection of class names, expressions of type  $\exists r.C$  and negations of lhs expressions
    - no closed classes!
- Example:  
 $\exists \text{married}.T \sqsubseteq \neg \text{Free} \sqcap \exists \text{has.Sorrow}$

# OWL 2 Reasoner

- OWL 2 DL:
  - Pellet <http://clarkparsia.com/pellet/>
  - Hermit <http://www.hermit-reasoner.com/>
- OWL 2 EL:
  - CEL <http://code.google.com/p/cel/>
- OWL 2 RL:
  - essentially any rule engine
- OWL 2 QL:
  - essentially any SQL engine (with a bit of query rewriting on top)

# References

- OWL 2 W3C Documentation
  - <http://www.w3.org/TR/owl2-overview/>
- Pascal Hitzler, Markus Krötzsch, Sebastian Rudolph, York Sure, Semantic Web – Grundlagen. Springer, 2008.  
<http://www.semantic-web-grundlagen.de/>
- Pascal Hitzler, Markus Krötzsch, Sebastian Rudolph, Foundations of Semantic Web Technologies. CRC Press, 2009.  
<http://www.semantic-web-book.org/>  
(Grab a flyer from us.)



**Thanks!**

**[http://semantic-web-grundlagen.de/wiki/ESWC09\\_Tutorial](http://semantic-web-grundlagen.de/wiki/ESWC09_Tutorial)**